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AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

INFLUENCE OF DIVERSION ON THE MISSISSIPPI AND ATCHAFALAYA RIVERS

BY E. F. SALISBURY,[1] M. AM. SOC. C. E.

SYNOPSIS

The effect of an outlet, operative at all stages, from the Mississippi River into the Atchafalaya River, is discussed in this paper. This continuous diversion decreases the volume of the Mississippi and increases that of the Atchafalaya. The result of this diversion is set forth and provides a means of proving or of disproving some of the present hydraulic theories and opinions.

For present purposes, a like volume of water (bank-full stage) through the various years, and the passage of this like volume of water past the gauge station as registered by the gauge height are analyzed at points on both rivers.

On the Mississippi River, gauge stations are analyzed where they are not affected, as well as where they are affected, by continuous diversion, in order to obtain comparative data and to place before the mind of the reader some of the ordinary causes that influence the variation in gauge height for the same volume of discharge. This analysis shows: (1) That at points not affected by diversion operative at all stages, the same volume of water is now passing at the same elevation as in earlier years; (2) that the Mississippi River has conformed itself to the hydraulic theory and increased its slope as the volume has been diminished by diversion; and, likewise (3), the Atchafalaya River has conformed itself to the hydraulic theory and flattened its slope as its volume has been increased through diversion.

INTRODUCTION

At only one point in the entire reach of the Mississippi River is its water diverted at practically all stages, namely, at Red River Landing (Angola, La.), about 190 miles up stream from New Orleans, where it is diverted into

NOTE.—Discussion on this paper will be closed in February, 1936, *Proceedings*.

[1] Chf. Engr., L. & A. Ry., Minden, La.

the Atchafalaya River. The result of this diversion is clearly reflected in the available data and should prove or disprove present theories and opinions.

In his study of sediment-bearing streams, Guglielinini, an Italian engineer of the Seventeenth Century, advanced the hydraulic theory[2] that: (1) The greater the quantity of water a stream carries, the greater will be its fall; (2) the greater the force of a stream, the less will be the slope of its bed; and (3), the slope of the bottom in rivers will diminish in the same proportion in which the body of water is increased, and *vice versa*.

Commenting on this theory, Humphreys and Abbot state[2]:

"* * *. These rules have their explanation in the facts, that the beds of rivers, of the character above mentioned, are capable of resisting, unchanged, only a certain velocity of current; and, on the other hand, that the sedimentary matter, contained in the river-water requires a certain degree of velocity to keep it in suspension. From the counteracting tendencies of the above two causes, a mean becomes established at which the current ceases to deposit its sediment, and the bottom ceases to be abraded; in other words, the bottom becomes permanent. But if, from any cause, such as throwing off a portion of the water through a waste-weir, the velocity of the current is diminished, it is no longer able to maintain its sediment in suspension, but will continue to deposit in its bed, until, through the elevation of the bed, its velocity again becomes, what it was before it was disturbed, sufficient to maintain its sediment in permanent suspension."

While he was a member of the Mississippi River Commission, the late James B. Eads, F. Am. Soc. C. E., clearly and ably expounded this same theory. In a letter to the Mississippi River Commission[3] dated April 12, 1882, he stated emphatically that, in flood time, the current cannot be checked in the slightest degree without causing a deposition of some part of the sediment. In this connection he named three controlling factors that are involved in every problem presented by the characteristic phenomenon of the Mississippi River: (1) The force of the current; (2) the frictional resistance of the river bed; and (3) the intimate relation between the quantity of sediment carried in the water, and the velocity of the current. If the current is increased or decreased from any cause (such as the friction between the water and the bed of the stream) the quantity of sediment carried in suspension is likewise increased or decreased. If the volume is diminished, the ratio of friction to volume will be increased; and, conversely, if the volume is increased the ratio of frictional resistance will be decreased.

Diagrammatically, Mr. Eads demonstrated the rapidity with which frictional resistance increases if the sedimentary river is divided into two or more channels. Unless the two new channels have steeper slopes it is impossible for the water to flow as fast as it would in the original, single channel. Quoting from the letter:

"There are no truths more capable of complete demonstration or more generally recognized by hydraulic engineers than these and the effects already treated by the Atchafalaya bear ample testimony to the soundness of

[2] "Report on the Physics and Hydraulics of the Mississippi River", by A. A. Humphreys and H. L. Abbot, 1861 (reprinted 1876), *Professional Papers No. 4*, Corps of Topographical Engrs., U. S. A., p. 415.

[3] Annual Rept. of the Chf. of Engrs., U. S. Army, to the Secretary of War, for the Year, 1882, Pt. III, Appendix RR, p. 2769 *et seq.*

the deductions thus advanced. For instance, the volume in the main river below the Atchafalaya outlet is decreased by the volume carried off through the Atchafalaya. Therefore, in proportion as the lower Mississippi has lost this volume the slope of the river or its flood line should be steepened. The volume has been steadily increased in the Atchafalaya and as a natural result of such increase its slope has diminished."

If the foregoing theory is correct, the action of the Mississippi River at the point of almost constant diversion is a logical place to test its accuracy. Opposite Red River Landing the Red and Atchafalaya Rivers are virtually parallel to the Mississippi River (see Fig. 1), with a connection between the two called "Old River." This junction has been ably compared to the capital letter, H. The right-hand side of the letter is the Mississippi River; the other side above the cross-bar, the Red River, and below the cross-bar, the Atchafalaya; the cross-bar connection, seven miles in length, is Old River. The direction of flow in Old River is dependent on stages prevailing on the Mississippi and Red Rivers, the flow being most frequently from the Mississippi River through Old River into the Atchafalaya. This condition can occur at all stages of the Mississippi. The volume of water diverted from the Mississippi into the Atchafalaya decreases the volume of water in the Mississippi River below Old River. Therefore, if the hydraulic theory is correct, the slope of the Mississippi River below Old River should be increased. Furthermore, as the volume in the Atchafalaya is increased by waters from the Mississippi, the slope of the Atchafalaya River should be diminished.

In testing the correctness of this hydraulic theory, it is necessary to follow a like volume of water through the various years and the passage of this like volume of water by the gauge stations as registered by the gauge heights. In selecting this like volume of water it should be close to the bank-full stage of the river. This selection will give a desirable wide range of gauge readings for consideration not only in the different years, but during the same year. Furthermore (and this is very important), it represents the flow at bank-full stage, the form of the river which all the forces acting on it during the year are striving to create. The usual mistake made by river experts is to confine their discussions to flood stages, ignoring the forces acting during low water. A flood acts but a comparatively short time and during the remainder of the 365 days the river modifies the results which then occur.

It is apparent from the discharge observations recorded by the Mississippi River Commission that the above requirements are fulfilled by a volume of 1 100 000 to 1 200 000 cu ft per sec for the Mississippi River, and 300 000 cu ft per sec for the Atchafalaya River. In preparing the study for each gauge station, all gauges published by the Mississippi River Commission and measured with meters were used, falling within 25 000 cu ft per sec above or below the selected volume for the stations on the Mississippi River and 15 000 cu ft per sec above or below the selected volume for the Atchafalaya River. The gauge for these volumes was adjusted to give a gauge for the selected volume. Readings measured with rods and floats were not used. All recorded readings were checked as to influence by crevasses and levee

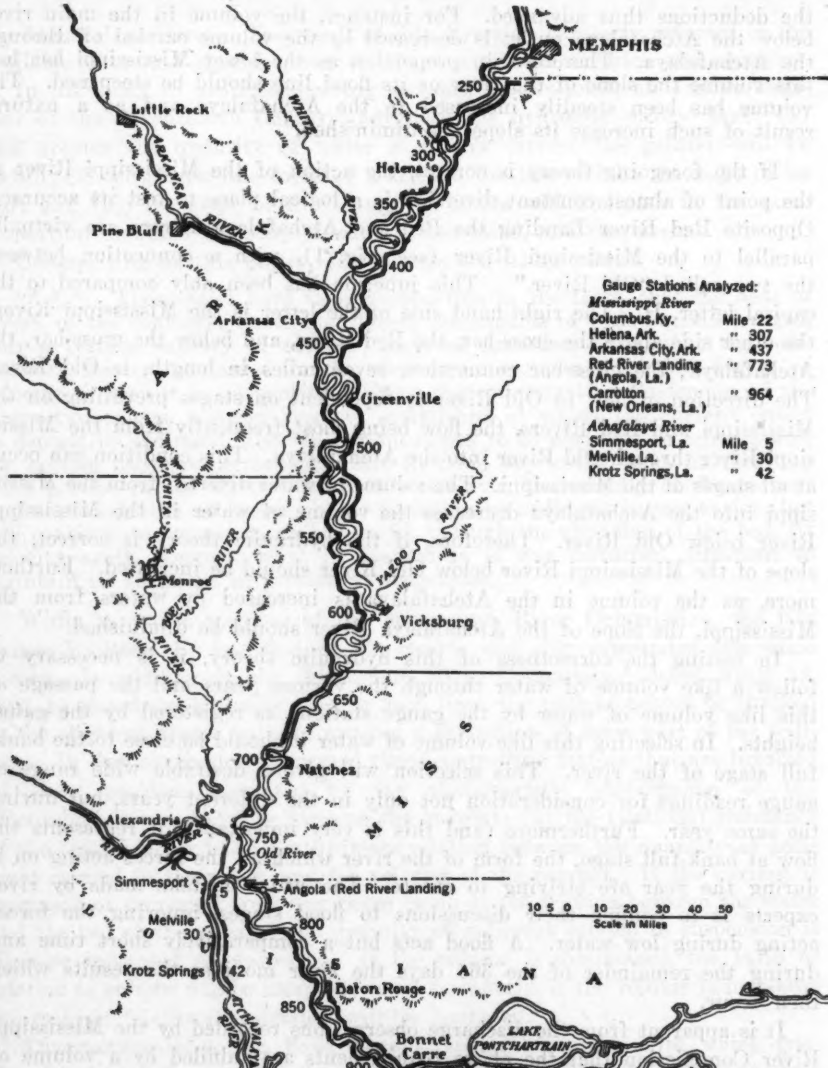
MEMPHIS
250
Little Rock
ARKANSAS
300
Helena
350
Pine Bluff
RIVER
400
Gauge Stations Analyzed:
Mississippi River
Columbus, Ky. Mile 22
Helena, Ark. " 307
Arkansas City, Ark. " 437
Red River Landing " 773
(Angola, La.)
Carrolton " 964
(New Orleans, La.)
Achafalaya River
Simmesport, La. Mile 5
Melville, La. " 30
Krotz Springs, La. " 42
Arkansas City
450
Greenville
500
550
YAZOO
RIVER
Monroe
600
Vicksburg
650
700
Natchez
Alexandria
RIVER
750
Old River
Simmesport
5
Angola (Red River Landing)
Melville
30
800
10 5 0 10 20 30 40 50
Scale in Miles
Krotz Springs
42
Baton Rouge
850
Bonnet
Carre
LAKE
PONTCHARTRAIN
900
950
NEW
ORLEANS
(Carrolton)
Caernavon
Morgan City
1000
Pointe La Hatche
1050
G U L F O F M E X I C O

FIG. 1

changes. These volumes are close to bank-full stage and no effect from crevasses was found. At only one point was the gauge station affected by levee changes and the results are discussed subsequently. That crevasses and levee confinement have not had any measurable effect on the carrying capacity of the main river is substantiated by reports of the Army Engineers.[4] During the period of this analysis only two cut-offs occurred; one at Mile 680 (Waterproof), in 1884; the other at Mile 638 (Yucatan), in 1928. These cut-offs are approximately 100 miles away from the nearest gauge station used in this paper and are so distant that they would not have any measurable effect.

To analyze the gauge and discharge record at any station it is necessary to understand that a silt-bearing river is somewhat human in its ability to do more work at one time than another. It will discharge a given volume of water at different gauge heights, depending upon its physical condition and the movement of water. This phenomenon of the river's capacity, to do more work at one time than another, as reflected by the passage of a given volume of water at different gauge heights is, for the purpose of this paper, termed its "efficiency."

It is generally recognized that the system used in taking discharge observations of large rivers, especially during high stages, is subject to error, but that it is sufficiently accurate for practical purposes. It is impossible to separate the factor of error in discharge measurements and the factor of river efficiency; both must be considered in river work. Therefore, it is consistent to embody the two in the term, "efficiency," which is so used in this analysis.

The variation in gauge heights for the same volume of water due to the efficiency of the river is clearly shown at gauge stations not influenced by unrestricted diversion; that is, uncontrolled diversion of water from the parent river at all stages, such as that from the Mississippi River into the Atchafalaya River. Three gauge stations selected for this purpose are those at Columbus-Wickliffe, Ky., and Helena, and Arkansas City, Ark. These three stations were selected over any of the other gauge stations because they furnish more readings of the selected volume. An analysis of a discharge of 1 200 000 cu ft per sec at these three stations is afforded by reference to Tables 1(*a*), 1(*b*), and 1(*c*). A summary of these data is given in Tables 2(*a*), 2(*b*), and 2(*c*).

Columbus, Kentucky, Gauge

All the discharges used were measured with current meters and those varying by more than 25 000 cu ft per sec from a volume of 1 200 000 cu ft per sec were rejected. Discharges measured with float or rod were not used in any of the studies of this paper. At the Columbus gauge (see Table 1 (*a*)), the readings were adjusted to a basis of 1 200 000 cu ft per sec, using 60 000 cu ft per sec per ft of gauge, as shown by the 1929 discharges. Only actual observed discharges for 1929 were used for the analysis of this record and no crevasses that would affect the readings were reported.

[4] "Spillways on the Lower Mississippi River", H. R. Doc. No. 95, 70th Cong., First Session, p. 4.

TABLE 1.—Study of Gauge Records

Date (1)	Gauge reading, in feet (2)	Discharge, in cubic feet per second (3)	Gauge reading,* in feet, adjusted to 1 200 000 cubic feet per second (4)
(a) Columbus (Ky.) Gauge, 22 Miles below Cairo, Ill. (Bank-full Stage, 40 Feet; Flood Stage, 43 Feet)			
April 15, 1892	41.2	1 203 000	41.2
March 3, 1897	39.2	1 188 000	39.4
February 1, 1898	40.5	1 228 000	40.0
February 3, 1898	40.1	1 181 000	40.4
February 4, 1898	39.6	1 177 000	40.0
March 29, 1907	41.4	1 213 000	41.2
April 24, 1911	41.1	1 176 000	41.5
March 3, 1923	43.0	1 219 000	42.7
March 24, 1923	42.8	1 213 000	42.6
April 29, 1929	42.5	1 174 000	42.9
April 30, 1929	43.0	1 191 000	43.2
(b) Helena (Ark.) Gauge, 307 Miles below Cairo, Ill. (Bank-full Stage, 39.0 Feet; Flood Stage, 44.0 Feet)			
April 6, 1882	43.3	1 210 000	43.1
May 3, 1892	44.5	1 209 000	44.3
May 4, 1892	44.6	1 206 000	44.5
June 2, 1892	44.7	1 194 000	44.8
June 3, 1892	44.7	1 177 000	45.1
June 4, 1892	44.6	1 180 000	45.0
June 6, 1892	44.6	1 185 000	44.9
June 7, 1892	44.6	1 177 000	45.0
June 8, 1892	44.6	1 198 000	44.6
June 9, 1892	44.7	1 203 000	44.7
May 9, 1893	43.1	1 190 000	43.3
May 10, 1893	43.3	1 210 000	43.1
May 11, 1893	43.5	1 197 000	43.5
March 25, 1908	45.1	1 228 000	44.6
March 12, 1929	42.9	1 195 000	43.0
March 13, 1929	43.7	1 219 000	43.4
May 3, 1929	45.9	1 222 000	45.5
May 4, 1929	45.9	1 218 000	45.6
(c) Arkansas City (Ark.) Gauge, 437 Miles below Cairo, Ill. (Bank-full Stage, 42 Feet; Flood Stage, 48 Feet)			
March 14, 1890	48.2	1 186 000	48.5
March 25, 1890	49.2	1 220 000	48.8
March 28, 1890	49.2	1 223 000	48.7
April 4, 1890	48.3	1 221 000	47.9
April 7, 1890	47.9	1 187 000	48.2
April 9, 1890	48.0	1 220 000	47.6
April 11, 1890	47.9	1 199 000	47.9
March 11, 1891	46.1	1 212 000	45.9
March 16, 1891	46.9	1 209 000	46.7
March 17, 1891	46.9	1 219 000	46.5
March 18, 1891	47.0	1 193 000	47.1
March 20, 1891	47.3	1 182 000	47.7
March 21, 1891	47.3	1 214 000	47.0
March 23, 1891	47.5	1 211 000	47.3
April 25, 1891	47.3	1 228 000	46.7
April 27, 1891	47.5	1 228 000	46.9
April 29, 1891	47.3	1 201 000	47.3
April 30, 1891	47.1	1 189 000	47.3
May 2, 1891	46.5	1 186 000	46.8
June 27, 1892	47.5	1 201 000	47.5
June 28, 1892	47.2	1 179 000	47.6
June 29, 1892	46.6	1 180 000	47.0
June 20, 1892	46.1	1 192 000	46.3
May 5, 1893	46.2	1 198 000	46.2
March 12, 1903	49.0	1 198 000	49.0
April 28, 1911	47.0	1 184 000	47.3
March 27, 1923	46.9	1 174 000	47.4
March 29, 1923	47.9	1 174 000	48.4
(c) Arkansas City (Ark.) Gauge.—(*Continued*)			
March 21, 1923	48.6	1 224 000	48.1
April 2, 1923	49.0	1 213 000	48.7
April 3, 1923	49.1	1 197 000	49.1
April 4, 1923	49.4	1 203 000	49.4
March 19, 1929	48.9	1 192 000	49.1
March 20, 1929	49.5	1 213 000	49.2
June 20, 1929	49.3	1 206 000	49.2
June 21, 1929	48.5	1 178 000	48.9
(d) Carrollton (La.) Gauge†, 964 Miles below Cairo, Ill. (Bank-full Stage, 9.0 Feet; Flood Stage, 17.0 Feet)			
April 7, 1883	15.4	1 086 000	15.6
February 2, 1885	13.5	1 102 000	13.5
March 4, 1890	14.9	1 110 000	14.7
March 7, 1890	15.2	1 128 000	14.7
March 14, 1890	Crevasse at Nita, Ky.		
March 22, 1890	15.9	1 124 000	15.5
March 28, 1890	15.5	1 106 000	15.4
March 31, 1890	15.2	1 082 000	15.5
April 3, 1890	15.6	1 106 000	15.5
April 4, 1890	15.4	1 092 000	15.5
April 5, 1890	15.3	1 102 000	15.3
April 7, 1890	15.3	1 096 000	15.4
April 8, 1890	15.2	1 091 000	15.4
March 4, 1891	14.4	1 100 000	14.4
March 9, 1891	14.6	1 093 000	14.7
March 10, 1891	14.9	1 114 000	14.7
March 12, 1891	15.3	1 103 000	15.2
May 18, 1892	16.2	1 083 000	16.5
May 19, 1892	16.1	1 111 000	15.9
June 4, 1892	16.6	1 091 000	16.8
June 6, 1892	16.7	1 088 000	16.9
June 8, 1892	16.7	1 121 000	16.3
June 10, 1892	17.0	1 079 000	17.4
June 12, 1892	Crevasse at Belmont, Ky., 911 miles below Cairo, Ill. Discharge, 140 000 cu ft per sec.		
June 14, 1892	16.7	1 092 000	16.8
June 19, 1893	17.0	1 089 000	17.2
June 21, 1893	17.2	1 120 000	16.9
June 22, 1893	17.3	1 113 000	17.1
June 23, 1893	17.5	1 106 000	17.4
June 26, 1893	17.0	1 089 000	17.2
May 3, 1898	15.6	1 096 000	15.7
May 4, 1898	15.5	1 087 000	15.7
April 23, 1904	15.7	1 091 000	15.9
February 2, 1907	17.1	1 091 000	17.2
February 5, 1907	17.4	1 100 000	17.4
February 6, 1907	17.5	1 090 000	17.7
February 7, 1907	17.6	1 114 000	17.4
March 29, 1909	15.9	1 110 000	15.7
March 30, 1909	16.2	1 093 000	16.3
March 31, 1909	16.2	1 107 000	16.1
April 1, 1909	16.4	1 095 000	16.5
April 2, 1909	16.4	1 102 000	16.4
April 3, 1909	16.4	1 100 000	16.4
April 4, 1909	16.4	1 108 000	16.3
April 5, 1909	16.4	1 089 000	16.6
April 7, 1909	16.4	1 118 000	16.1
April 8, 1909	16.5	1 089 000	16.7
April 9, 1912	16.5	1 091 000	16.7
April 10, 1912	16.8	1 110 000	16.6
April 11, 1912	17.2	1 104 000	17.1
February 12, 1913	16.0	1 095 000	16.1
February 13, 1913	16.2	1 099 000	16.2
February 14, 1913	16.4	1 099 000	16.4
February 15, 1913	16.4	1 099 000	16.4

* Except as noted in the case of Table 1 (*e*).

† Gauge reading, Column (4), adjusted to 1 100 000 cu. ft. per sec.

TABLE 1.—(*Continued*).

Date (1)	Gauge reading, in feet (2)	Velocity, in feet per second (3)	Discharge, in cubic feet per second (4)	Gauge reading,* in feet, adjusted to 1 200 000 cubic feet per second (5)
(*d*) CARROLLTON (LA.) GAUGE.—(*Continued*)				
February 16, 1913	16.5	....	1 091 000	16.7
February 17, 1913	16.6	....	1 085 000	16.9
February 18, 1913	16.7	....	1 084 000	17.1
February 19, 1913	17.0	....	1 101 000	17.0
February 20, 1913	17.0	....	1 088 000	17.2
February 21, 1913	17.0	....	1 112 000	16.8
February 23, 1913	17.0	....	1 106 000	16.9
February 24, 1913	17.0	....	1 112 000	16.8
February 25, 1913	16.9	....	1 087 000	17.1
February 26, 1913	16.7	....	1 074 000	17.1
April 20, 1913	17.9	....	1 124 000	17.5
April 22, 1913	18.3	....	1 124 000	17.9
April 29, 1913	18.8	....	1 109 000	18.6
May 6, 1913	19.0	....	1 118 000	18.7
May 18, 1913	18.5	....	1 102 000	18.5
May 19, 1913	18.2	....	1 080 000	18.5
May 20, 1913	17.9	....	1 089 000	18.1
February 1, 1916	16.6	....	1 123 000	16.2
April 27, 1917	16.6	....	1 111 000	16.4
April 23, 1920	17.6	....	1 072 000	18.1
April 24, 1920	17.7	....	1 086 000	17.9
April 25, 1920	17.8	....	1 085 000	18.1
April 4, 1922	17.3	....	1 073 000	17.8
April 8, 1922	18.6	....	1 091 000	18.8
April 11, 1922	19.3	....	1 128 000	18.8
April 11, 1929	17.3	....	1 109 000	17.1
April 12, 1929	17.4	....	1 078 000	17.8
April 15, 1929	17.7	....	1 102 000	17.7
April 16, 1929	17.7	....	1 110 000	17.5
April 18, 1929	17.7	....	1 108 000	17.6
April 19, 1929	17.7	....	1 097 000	17.8
April 20, 1929	18.0	....	1 124 000	17.6
April 23, 1929	18.2	....	1 118 000	17.9
April 24, 1929	18.3	....	1 111 000	18.1
April 26, 1929	18.3	....	1 122 000	17.9
April 30, 1929	18.3	....	1 118 000	18.0
June 26, 1929	18.4	....	1 108 000	18.3
February 8, 1932	16.7	....	1 104 000	16.6
February 10, 1932	17.0	....	1 091 000	17.2
February 11, 1932	17.1	....	1 107 000	17.0
February 16, 1932	17.6	....	1 117 000	17.3
March 16, 1932	17.4	....	1 092 000	17.5
(*e*) SIMMESPORT (LA.) GAUGE‡, 5 MILES BELOW THE HEAD OF ATCHAFALAYA RIVER.				
April 7, 1890	42.3	5.33	306 594	41.8
March 23, 1891	40.9	4.69	296 482	41.2
March 25, 1891	41.0	4.68	301 000	41.0
March 28, 1891	40.9	4.61	293 244	41.4
March 30, 1891	41.1	4.70	296 996	41.3
April 4, 1891	41.1	4.71	302 113	40.9
April 6, 1891	41.1	4.80	307 522	40.5
April 9, 1891	41.2	4.69	302 555	41.0
April 15, 1891	41.4	4.76	306 075	40.9
April 17, 1891	41.4	4.72	300 510	41.4
May 20, 1892	41.7	5.10	303 049	41.5
May 22, 1892	41.8	4.93	296 983	42.0
May 24, 1892	41.9	5.07	302 257	41.7
May 26, 1892	41.9	4.96	299 479	41.9
May 28, 1892	42.0	5.11	306 556	41.5
February 14, 1907	41.5	4.92	299 945	41.5
February 16, 1907	41.9	4.90	289 262	42.8
April 6, 1908	39.9	5.04	290 389	40.7
May 4, 1908	42.1	5.08	299 270	42.1
May 6, 1908	42.3	5.15	310 578	41.5
June 16, 1908	44.9	4.84	312 262	43.9
April 13, 1912	43.4	4.95	290 348	44.2
April 24, 1920	43.8	4.49	298 119	44.0
April 27, 1920	44.3	4.41	297 512	44.5
May 3, 1920	45.1	4.50	305 979	44.7
April 13, 1923	42.7	4.69	311 061	41.8

* Except as noted in the case of Table 1 (*e*).

‡ Gauge reading, Column (4) adjusted to 300 000 cu ft per sec.

TABLE 1—(*Continued*).

Date (1)	Gauge reading, in feet (2)	Velocity, in feet per second (3)	Discharge, in cubic feet per second (4)	Gauge reading,* in feet, adjusted to 1 200 000 cubic feet per second (5)
(*e*) Simmesport (La.) Gauge‡.—(*Continued*)				
July 17, 1928	37.8	4.40	287 379	38.8
July 21, 1928	37.8	4.39	290 914	38.6
April 6, 1929	39.9	4.29	301 995	39.9
April 10, 1929	40.5	4.42	310 596	39.7
April 21, 1933	37.9	4.34	290 457	38.6
April 22, 1933	38.8	4.39	298 232	39.0
April 24, 1933	39.0	4.33	299 665	39.0
April 26, 1933	39.3	4.37	297 111	39.5
April 27, 1933	39.6	4.34	302 736	39.4
May 1, 1933	40.1	4.48	315 364	39.0
May 3, 1933	40.6	4.50	315 306	39.5
May 5, 1933	40.9	4.43	315 515	39.8
May 13, 1933	41.0	4.45	311 273	40.1

* Except as noted in the case of Table 1 (*e*).
‡ Gauge reading, Column (4) adjusted to 300 000 cu ft per sec.

TABLE 2.—Comparative Gauge Variation for a Discharge* of 1 200 000 Cubic Feet per Second

Maximum† variation for discharging the same volume of water during: (1)	Period of record, in years (2)	Gauge Heights, in Feet: Lowest: Date (3)	Lowest: Reading (4)	Highest: Date (5)	Highest: Reading (6)	Variation, in feet (7)
(*a*) Columbus-Wickliffe (Ky.) Gauge Station						
The earliest and latest recorded year‡	37	March 24, 1929	42.9	April 15, 1892	41.2	1.7
Any one year	37	April 29, 1929	42.9	April 30, 1929	43.2	0.3**
The 37-year period	37	March 3, 1897	39.4	April 30, 1929	43.2	3.8
(*b*) Helena (Ark.) Gauge Station						
The earliest and latest recorded year ‡	47	March 12, 1929	43.0	April 6, 1882	43.1	0.1
Any one year	47	March 12, 1929	43.0	May 4, 1929	45.6	2.6
The 47 year period	47	March 12, 1929	43.0	May 4, 1929	45.6	2.6
(*c*) Arkansas City (Ark.) Gauge Station						
The earliest and latest recorded year ‡	39	June 21, 1929	48.9	March 25, 1890	48.8	0.1
Any one year	39	March 27, 1923	47.4	April 4, 1923	49.4	2.0
The 39-year period	39	May 5, 1893	46.2	April 4, 1923	49.4	3.2
(*d*) Red River Landing (Angola, La.) Gauge Station§						
The earliest and latest recorded year	50	February 17, 1932	49.0	June 20, 1882	42.1	6.9
Any one year	50	February 15, 1913	45.9	May 5, 1913	50.4	4.5
The 50-year period	50	January 21, 1885	40.2	May 5, 1913	50.4	10.2
(*e*) Carrollton (La.) Gauge Station§						
The earliest and latest recorded year	49	February 8, 1932	16.6	April 7, 1883	15.6	1.0
Any one year	49	February 12, 1913	16.1	May 6, 1913	18.7	2.6
The 49-year period	49	February 2, 1885	13.5	April 6, 1922	18.8	5.3
(*f*) Simmesport (La.) Discharge Observation Station, on the Atchafalaya River‖						
The earliest and latest recorded year	43	April 7, 1890	41.8	May 13, 1933	40.1	1.7
Any one year	43	April 8, 1908	40.7	June 16, 1908	43.9	3.2
The 43-year period	43	June 21, 1928¶	38.6	May 3, 1920	44.7	6.1

* Except as noted in the case of Tables 2(*d*), 2(*e*), and 2(*f*).
† Except as noted.
‡ Minimum.
§ For a discharge of 1 100 000 cu ft per sec.
‖ For a discharge of 300 000 cu ft per sec.
¶ Also April 21, 1933.
** Sufficient readings not available to determine maximum variation during the same flood

Referring to Table 2 (*a*), the highest gauge of 1892 is 1.7 ft lower than the lowest gauge of 1929. This indicates that the two different floods, although rated at two different "efficiencies" at this station, were at one time discharging the same volume of water at the same gauge height. In other words, between 1892 and 1929, no appreciable silting occurred at this station. The limited number of observations during any one year at this station (Table 2 (*a*)) does not reflect the gauge variation that can be expected during a flood, due to "efficiency." For a 37-yr period, the greatest range on the gauge to discharge the same volume of water, is 3.8 ft (see Table 2 (*a*)) which is a measure of the effect of "efficiency" at this station, for all floods recorded (see Table 1 (*a*)).

Helena, Arkansas, Gauge

Referring to the Helena gauge (Table 2 (*b*)), 55 000 cu ft per sec per ft of gauge was used in adjusting readings to a discharge of 1 200 000 cu ft per sec. As in the case of the Columbus-Wickliffe gauge only actual observed discharges during 1929 were used.

Crevasses occurring within 50 miles above and below Helena prior to the discharge rates listed in Table 1 (*b*), in general did not affect the recorded gauges. Records of crevasses that may have occurred in 1882 were not available to the writer. The recorded gauges of 1893 that were not affected by crevasses, however, are practically the same as those for 1882. From this evidence and the fact that the discharge is close to bank-full stage, the discharge of 1882 is considered to be unaffected by crevasses. In other words, it would not affect the analysis if the discharges of 1882 or 1893 were used.

Reference to Table 2 (*b*) demonstrates that, in the two different floods, 47 yr apart, the same volume of water was passing at the same gauge height, which indicates that there has been no filling or silting in the channel at this station. The greatest variation in discharging the same quantity of water occurred in 1929. The difference of 2.6 ft between the lowest and highest gauge of this year reflects the effect of efficiency during the same flood. For the 47-yr period, the greatest range on the gauge for discharging 1 200 000 cu ft per sec is 2.6 ft, which value measures the influence of "efficiency" at the Helena gauge for all the floods recorded. It is to be noted (see Table 2 (*b*)) that the lowest and highest gauges of any of the years recorded occurred in 1929.

Arkansas City, Arkansas, Gauge

Effect of Crevasses on the Gauge Study.—Prior to the discharge reading of March 14, 1890 (see Table 1 (*c*)), there was a crevasse of 35 000 cu ft per sec, 8 miles above the Arkansas City gauge on March 9, 1890, and another of 6 055 cu ft per sec, 35 miles below on March 7, 1890 (see Table 3). These crevasses were too small to affect the gauge reading as shown by the increase of 0.3 ft in the gauge height by March 25, 1890. In other words, the gauge height increased despite the occurrence of two additional small crevasses (19 901 cu ft per sec) 4 miles above, and two of 35 000 and 31 000 cu ft per sec,

7 and 40 miles, respectively, below the Arkansas City gauge. Prior to the gauge reading of April 11, 1890 (see Table 1 (*c*)), in the 50-mile reach above this gauge, nine crevasses occurred, discharging 515 901 cu ft per sec; and, in the reach 50 miles below, six crevasses occurred, having a discharge of 242 255 cu ft per sec, or a total of 758 156 cu ft per sec. A comparison of the gauge of April 11, 1890, with that of March 14, 1890, shows a reduction of only 0.6 ft to discharge the same volume of water (1 200 000 cu ft per sec) with more than 700 000 cu ft per sec of water diverted through crevasses within 50 miles above and 50 miles below the gauge. This small variation proves that there was little, if any, effect from crevasses, which conclusion is further supported by a comparison of the gauges of 1890 and 1891.

TABLE 3.—Record of Crevasse Occurrence

Item	Date	Miles below Cairo, Ill.	Bank	Discharge, in cubic feet per second
(1)	(2)	(3)	(4)	(5)
	(*a*) Vicinity of Arkansas City Gauge (437 Miles below Cairo)			
1	March 9, 1890..	429	Right	35 000
2	March 7, 1890..	472	Right	6 055
	Total, prior to March 14, 1890	.....	.....	41 055
3	March 24, 1890..	433	Right	17 000
4	March 24, 1890..	433	Right	2 901
5	March 18, 1890..	444	Left	35 000
6	March 18, 1890..	467	Right	31 000
	Total, prior to March 25, 1890	.....	.....	85 901
7	March 27, 1890..	426	Right	17 000
8	March 27, 1890..	432	Right	10 000
	Total, prior to March 28, 1890	.....	.....	27 000
9	April 5, 1890....	426	Right	44 000
10	April 4, 1890....	432	Left	208 000
11	April 4, 1890....	431	Left	114 000
12	March 28, 1890..	435	Left	68 000
13	March 28, 1890..	438	Left	125 000
14	March 28, 1890..	470	Right	35 200
15	April 7, 1890....	474	Left	10 000
	Total, prior to April 1, 1890..	.....	.....	604 200
	Total, in 1890...	.....	.....	758 156
16	May 13, 1892...	452	Right	91 623
17	May 25, 1892...	484	Right	17 253
18	June 2, 1892....	439	Right	19 554
19	June 22, 1892...	470	Right	18 483
	Total in 1892, prior to July 1	.....	.....	146 913
	(*b*) Vicinity of Red River Landing Gauge (773 Miles below Cairo, Ill.)			
20	March 14, 1884..	789	Right	211 000
21	March 14, 1890..	899	Left	....
22	March 16, 1891..	961	Right	91 000
23	March 14, 1903..	738	.....	....
	(*c*) Vicinity of Carrollton Gauge (964 Miles below Cairo, Ill.)			
24	March 14, 1890..	902	.....	402 000
25	March 16, 1891..	961	Right	91 000
26	....	939	.....	116 000

Prior to July 1, 1892 (see Table 3 (*a*)), four crevasses, occurred within a reach of 50 miles below the Arkansas City gauge, with a total discharge of 146 913 cu ft per sec. The recorded readings in this year were not appreciably more or less than those for years that were not affected by crevasses.

The highest gauge of 1890 (see Table 2 (*c*)) is 0.1 ft lower than the lowest gauge of 1929, showing that at one time the two different floods were discharging the same volume of water at practically the same gauge. Again, this shows that no appreciable silting had occurred at this station in the 39-yr period. For the same discharges, the year 1923 showed the greatest variation in gauge heights. The difference of 2.0 ft (see Table 2 (*c*)) between the highest and lowest gauge of this year, for an adjusted discharge of 1 200

cu ft per sec, reflects the effect of "efficiency" during the same flood. For the entire 39-yr period, the greatest range on the gauge for discharging this same volume of water is 3.2 ft, which, likewise, measures the influence of "efficiency" at the Arkansas City gauge for all the floods on record.

SUMMARY: COLUMBUS-WICKLIFFE, HELENA, AND ARKANSAS CITY, GAUGES

Flood Efficiency.—The foregoing data show that the maximum variation is 2.6 ft for the same volume of water during the same flood at these three gauge stations. A variation in flood "efficiency" of approximately 3 ft can be expected at these stations. Floods divide themselves into the following three classes: (1) Those of poor "efficiency" (high gauge height for a given discharge); (2) those of normal "efficiency" (medium gauge height for a given discharge); and (3) those of high "efficiency" (low gauge height for a given discharge). Certain of the ordinary causes that influence the class of each flood for the same volume of discharge are: (*a*) The relationship between the velocity of the main stream and the sediment introduced by tributaries; (*b*) control of sediment carried by tributary streams; (*c*) channel stabilization on the entire river system; (*d*) extent to which the main channel has been flushed by intervening water stages; (*e*) the effect of winds in the direction of flow; (*f*) the effect, on a gauge in the main river close to the mouth of a tributary stream, of the discharge of a large volume of water; and (*g*) change in slope for rising and falling stages of the river.

There is a relationship between velocity and quantity of sediment carried (Cause (*a*)). If the velocity of the main river, required for carrying sediment, is not overtaxed by the sediment contributed by the tributaries the deficiency is picked up from the channel of the main river, causing lower gauge heights. On the other hand, if the sediment contributed is greater than can be transported by the parent river, it is deposited and causes a higher gauge. As for Cause (*b*), an attempt to control the sediment carried by the tributaries, in order to assist the parent river, by protecting it against erosion is advisable. This policy has been adopted, to some extent, for reasons far removed from flood-control purposes, such as the terracing of cultivated fields in hill farms required by the Federal Land Banks and vigorously pursued by the United States Department of Agriculture in erosion control and forest protection. Channel stabilization by protecting the banks on both the main river and the tributaries (Cause (*a*)) is also an aid in reducing the quantity of sediment. Prevailing winds in the direction of flow (Cause (*e*)) will lower the gauges for the same discharge, whereas prevailing winds in the opposite direction will raise them The change in rate of rising stages over falling stages (Cause (*g*)) affects the slope at corresponding gauge heights thereby producing different volumes of discharge. For slow falling stages the slope is flattened out from that produced by fast rising stages.

The extent to which Causes (*a*) to (*g*) act in unison controls, to a major degree, the class "efficiency" of the flood. It is not unusual for a combination of the adverse causes to occur at the same time, creating a flood of poor "efficiency."

The greatest variation at the three gauge stations between the earliest and latest flood-years is 1.7 ft, showing that the two floods are of different efficiency. Each flood, although 37 yr apart, passed the same volume of water at one time during the flood interval with a difference of 1.7 ft. As this is within the expected range of maximum gauge variation, due to the influence of "efficiency," the river is not silting up at these gauge stations and causing higher flood elevations. This is further substantiated by the cross-sections taken at various intervals by the Mississippi River Commission. Comparison of cross-section elements of the general survey of 1894 to 1904 with those of the survey of 1911 to 1913 covering the section of the river from Cairo to Red River shows, at bank-full stage, the average area increased 2.3%, the average width increased 3.9%, and the average mean depth increased 1.3 per cent.[5]

The levee systems at these gauge stations have been improved gradually by raising and strengthening. These levee changes have not affected the gauge readings recorded. As will be demonstrated subsequently for the Atchafalaya River the gauge height for a given volume of water without levees is increased after levee confinement at some point; however, after once confined, further levee extension down stream from this point does not influence the gauge for the same volume of water.

Overbank Diversion.—During 1921 the Cypress Creek gap in the levee line was closed immediately above Arkansas City. Previously, flood water did escape from the Mississippi River through this gap at overbank elevations. Subsequent to this date, flood waters have been confined and could only escape by crevassing. The overbank flood-water escape of this frequency did not have any noticeable effect on the efficiency of the Arkansas City gauge.

Red River Landing (Angola, La.) Gauge

The gauge station at Red River Landing is situated on the Mississippi River approximately 1 mile below Old River (see Fig. 2). All the discharges used (see Table 4) were those measured with current meters. Those varying more than 25 000 cu ft per sec from a volume of 1 100 000 cu ft per sec, were rejected. At this gauge, the readings were adjusted to 1 100 000 cu ft per sec, using 35 000 cu ft per sec per ft of gauge, as shown by the 1929 discharge, and only the actual discharges during that year were used.

The major crevasses that occurred within 50 miles above or below Red River Landing prior to the discharge dates in Table 4, are given in Table 3. Data concerning crevasse occurrence in 1882 were not available. The Morganza crevasse, 16 miles below the gauge station at Red River Landing, which occurred on April 16, 1874, was not closed until February, 1884. It breached again on March 14, 1884, and was closed in January, 1887; breached once more on April 22, 1890, was closed in March, 1891, and has remained closed since.[6] The levee breach at Morganza was open during the recorded

[5] Basic Data, Mississippi River, Annex No. 5, H. R. Doc. No. 798, pp. 71–73.
[6] Rept of the Mississippi River Comm., 1894, p. 3067.

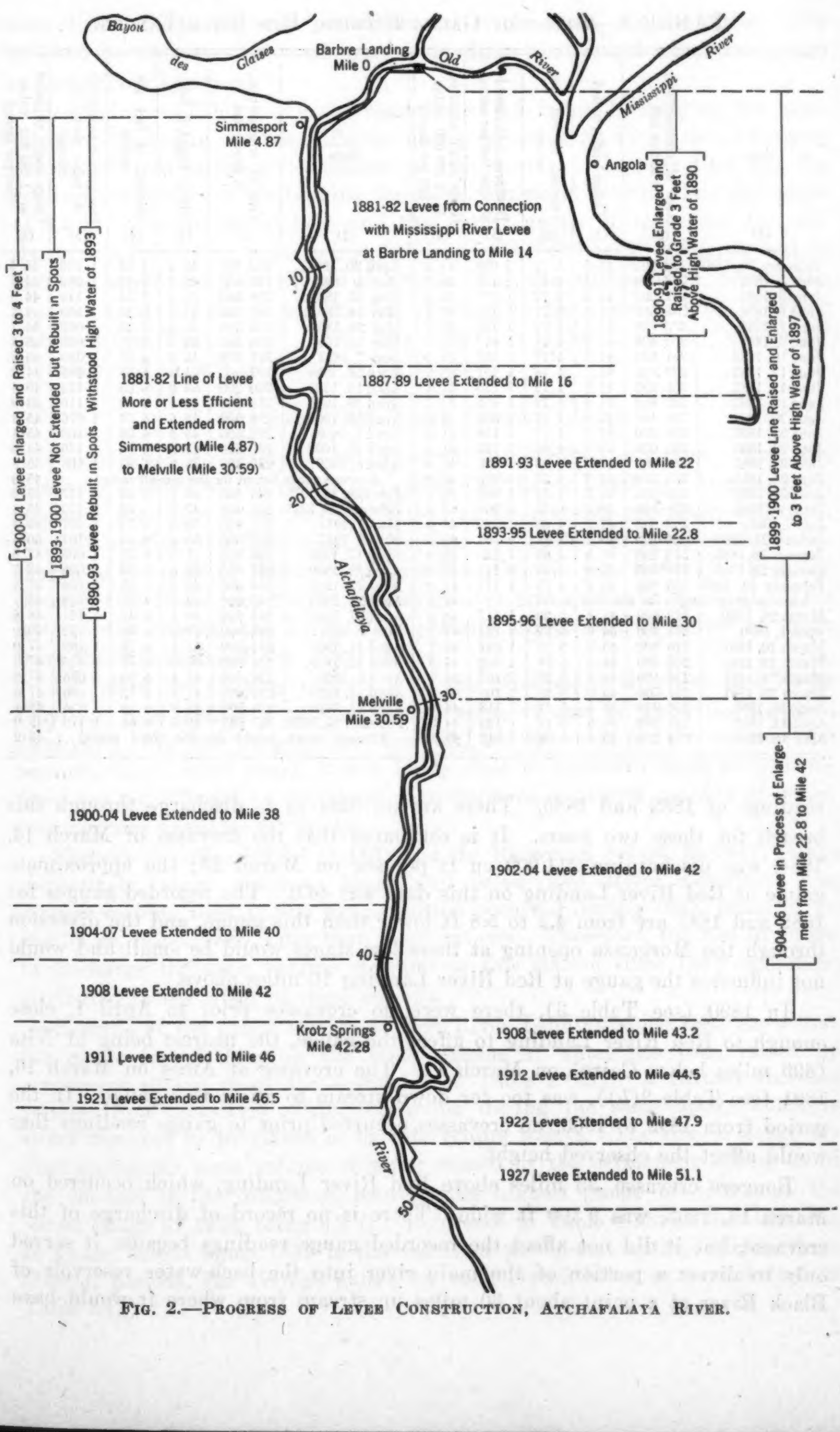

FIG. 2.—PROGRESS OF LEVEE CONSTRUCTION, ATCHAFALAYA RIVER.

TABLE 4.—STUDY OF GAUGE RECORDS, RED RIVER LANDING

Date (1)	Area of cross-section, in square feet (2)	Gauge height, in feet (3)	Velocity, in feet per second (4)	Discharge, in thousands of cubic feet per second (5)	Gauge height adjusted to 1 100 000 cubic feet per second (6)
February 10, 1882	192 200	41.5	5.72	1 099	41.5
June 3, 1882.....	233 100	41.4	4.83	1 125	40.7
June 5, 1882.....	232 200	41.5	4.72	1 097	41.5
June 6, 1882.....	233 500	41.5	4.77	1 114	41.1
June 7, 1882.....	233 900	41.5	4.79	1 120	40.9
June 8, 1882.....	232 400	41.6	4.81	1 117	41.1
June 10, 1882....	234 500	41.6	4.77	1 119	41.1
June 12, 1882....	233 800	41.7	4.74	1 107	41.5
June 15, 1882....	236 300	41.8	4.72	1 115	41.4
June 19, 1882....	233 800	41.8	4.72	1 105	41.7
June 20, 1882....	234 400	41.8	4.67	1 094	42.0
June 21, 1882....	234 200	41.8	4.77	1 118	41.3
June 23, 1882....	234 000	41.8	4.80	1 123	41.1
June 24, 1882....	234 100	41.8	4.68	1 096	41.9
June 26, 1882....	235 200	41.8	4.67	1 099	41.8
June 28, 1882....	235 400	41.7	4.61	1 085	42.1
June 30, 1882....	234 100	41.6	4.73	1 106	41.4
July 1, 1882......	232 800	41.5	4.68	1 089	41.8
January 21, 1885..	226 700	40.2	4.86	1 101	40.2
January 26, 1885..	229 300	41.5	4.88	1 120	40.9
January 29, 1885..	228 700	41.8	4.93	1 127	41.0
February 10, 1885	235 300	41.4	4.73	1 113	41.0
Average gauge height for the first period........					41.3
March 28, 1890...	210 500	44.8	5.29	1 117	44.3
April 1, 1890.....	213 900	44.9	5.22	1 121	44.3
March 10, 1891...	210 800	43.2	5.13	1 082	43.7
March 16, 1891...	209 800	44.5	5.34	1 100	44.5
March 18, 1891...	210 100	44.8	5.23	1 102	44.7
March 20, 1891...	211 700	45.0	5.23	1 109	44.7
April 10, 1891....	216 900	45.4	5.14	1 118	44.9
April 13, 1891....	219 200	45.5	5.12	1 125	44.8
April 16, 1891....	218 200	45.4	5.08	1 112	45.1
April 20, 1891...	222 700	45.4	4.93	1 100	45.4
May 4, 1892....	199 600	43.8	5.42	1 082	44.3
May 23, 1892...	208 800	45.4	5.34	1 119	44.9
May 24, 1893...	207 900	44.3	5.27	1 100	44.3
May 26, 1893...	208 200	44.4	5.20	1 089	44.7
May 29, 1893...	208 200	44.5	5.11	1 070	45.4
June 7, 1893....	221 200	45.0	4.90	1 089	45.3
April 28, 1898...	192 600	44.3	5.63	1 084	44.8
April 10, 1903...	220 200	50.0	5.05	1 122	49.4
April 28, 1906...	219 500	44.4	5.08	1 116	43.9
April 30, 1906...	218 000	44.6	4.94	1 076	45.3
April 7, 1908....	222 500	45.3	4.98	1 108	45.1
April 13, 1908...	230 600	45.3	4.88	1 125	44.6
April 8, 1909....	237 100	45.9	4.63	1 103	45.8
Average gauge height for the second record......					45.0
February 15, 1913	245 800	46.4	4.53	1 117	45.9
February 20, 1913	243 100	47.0	4.60	1 122	46.4
May 2, 1913....	225 800	50.0	4.72	1 107	49.8
May 5, 1913....	227 500	50.4	4.65	1 101	50.4
April 17, 1920...	246 000	47.9	4.36	1 099	47.9
April 21, 1920...	241 600	48.6	4.39	1 098	48.7
April 22, 1920...	244 600	48.8	4.28	1 109	48.5
April 23, 1920...	243 500	49.0	4.39	1 110	48.7
April 27, 1920...	244 400	49.4	4.42	1 121	48.8
May 4, 1920....	243 000	49.6	4.44	1 122	49.0
April 11, 1923...	231 500	47.1	4.74	1 102	47.0
April 16, 1923...	236 800	48.0	4.73	1 126	47.3
April 8, 1929....	234 400	47.4	4.60	1 085	47.8
April 11, 1929...	232 400	47.8	4.72	1 106	47.6
April 18, 1929...	233 500	48.8	4.76	1 124	48.1
February 17, 1932	257 580	49.5	4.33	1 119	49.0
Average gauge height for the third period...					48.2

readings of 1882 and 1885. There are no data as to discharge through this breach for these two years. It is estimated that the crevasse of March 14, 1884, was discharging 211 000 cu ft per sec on March 25; the approximate gauge at Red River Landing on this date was 46.0. The recorded gauges for 1882 and 1885 are from 4.2 to 5.8 ft lower than this gauge, and the diversion through the Morganza opening at these low stages would be small and would not influence the gauge at Red River Landing 16 miles above.

In 1890 (see Table 3), there were no crevasses prior to April 1, close enough to Red River Landing to affect the gauge, the nearest being at Nita (899 miles below Cairo) on March 14. The crevasse at Ames on March 16, 1891 (see Table 3(*b*)), was too far down stream to affect this gauge. In the period from 1892 to 1903, no crevasses occurred prior to gauge readings that would affect the observed height.

Bougere crevasse, 35 miles above Red River Landing, which occurred on March 14, 1903, was 9 300 ft wide. There is no record of discharge of this crevasse, but it did not affect the recorded gauge readings because it served only to divert a portion of the main river into the back-water reservoir of Black River at a point about 30 miles up stream from where it would have

been diverted into the reservoir at the end of the levee. In the period, 1904 to 1932, no crevasses occurred prior to readings that would affect the gauge at Red River Landing.

Referring to Table 2 (*d*), the variation of 6.9 ft for discharging the same volume of water in the two different floods, 50 yr apart, shows that excessive silting occurred in the river channel at this station. The year 1913 had the greatest variation for discharging the same volume of water. The difference of 4.5 ft between the highest and the lowest gauge of this year for discharging 1 100 000 cu ft per sec is due in part to "efficiency" influence and in part to conditions created by diversion. For the entire 50-yr period the greatest range in gauge heights for discharging the same volume of water is 10.2 ft (see Table 2 (*d*)). This large variation is due in part to "efficiency" influence and in part to silting in the channel.

SUMMARY: RED RIVER LANDING GAUGE

Unrestricted Diversion.—The effect of unrestricted diversion from the Mississippi River at the gauge station at Red River Landing (Angola, La.), creates a condition at variance with that at the stations previously analyzed. At this location the Mississippi River has two outlets of levee-confined discharge to the Gulf of Mexico; one past New Orleans into the Gulf, the other through Old River into the Atchafalaya River and thence to the Gulf. This latter outlet through the Atchafalaya permits unrestricted diversion of water from the Mississippi River at all stages.

To follow the changes occurring in the Mississippi River at Red River Landing, it is necessary to consider the same volume of water during each flood-year. For this purpose, a volume of 1 100 000 cu ft per sec was selected because, in the early years, it was fairly close to bank-full stage and, therefore, was not affected by crevasses. Furthermore, a greater number of gauge readings was available for study.

Shreve's cut-off occurred in 1831 and the Raccourci cut-off, in 1847, being, respectively, short distances above and below Red River Landing. The earliest reading used at this gauge station occurred during 1882, being 51 yr and 35 yr, respectively, later than these cut-offs. Therefore, the rise in gauge to discharge the same volume of water at this station since 1882 cannot be attributed to the effect of these cut-offs.

Records of the gauge station at Red River Landing since 1882 show that the gauge for discharging the same volume of water has been raised by permanent silting in of the channel (see Table 2 (*d*)). A further increase results from temporary silting occurring during the same flood, which is either removed by that flood or by intervening waters. The rise in gauge for discharging the same volume of water caused by the permanent silting in of the channel is the difference between the highest gauge of 1882 and the lowest gauge of 1932, being 6.9 ft. Reference to Table 4 shows that this rise in gauge height since 1882 is the result of permanent silting during three distinct periods, the average gauge for each period being as summarized in Table 5 (*a*).

The greatest gauge variation of 4.5 ft (see Table 2 (*d*)) for discharging the same volume of water during any one year occurred in 1913. This variation is greater than would be expected normally as the flood of 1913 is the end of the second period and the beginning of the third period of silting transition. The low gauges that occurred during the first days of the 1913 flood have never been repeated. The high gauges recorded during the latter days of the 1913 flood became normal for the third period. Since 1913, the lowest gauge for discharging 1 100 000 cu ft per sec is 47.1 as against 45.9 during the first part of 1913, and the highest gauge is 50.4. Therefore, the range of "efficiency" is from 47.1 to 50.4 ft on the gauge, or 3.3 ft.

TABLE 5.—AVERAGE GAUGE HEIGHTS, GAUGE AT RED RIVER LANDING

Period No.	(*a*) INCREASE IN GAUGE HEIGHTS			(*b*) PEAK-FLOOD GAUGE HEIGHTS		
	Years	Average gauge, in feet, for a discharge of 1 100 000 cubic feet per second	Gauge increase, in feet	Years	Average peak-flood gauge, in feet	Increase for the period, in feet
1....	1882 to 1885	41.3	...	1867 to 1886	45.1	...
2....	1890 to 1909	45.0	3.7	1890 to 1907	48.7	3.6
3....	1913 to 1932	48.2	3.2	1912 to 1933	52.4	3.7

The greatest gauge variation for discharging the same volume of water is 10.2 ft for the 50-yr period of which 6.9 ft is the loss due to permanent silting and 3.3 ft reflects the influence of efficiency. A report of the Mississippi River Commission records the peak-flood gauges at Red River Landing.[7] The average of these peak-flood gauges, grouped as three periods in Table 5 (*a*), gives the increase in average peak floods for each of these three periods (see Table 5 (*b*)). This average period peak-flood gauge of 7.3 ft is practically the same gauge increase as the 6.9-ft increase in gauge height for discharging the same volume of water, showing a direct influence of flood height on the loss in discharge capacity below the point of diversion.

The actual changes in cross-section elements for the 2-mile reach below Old River, in the period, 1882 to 1924, are, as follows[8]: The bank-full width increased 535 ft; the bank-full area decreased 13 089 sq ft; and the bank-full mean depth decreased 9.6 ft.

A peak-flood height of 50.5 ft was never exceeded during the periods prior to 1912. During the period, 1912–1933, this gauge was exceeded eight times with the maximum peak gauge of 57.5 ft, which occurred in 1927. The main river at this point has adjusted itself to an average peak-flood gauge of 52.4 ft.

Loss in the cross-sectional area at Red River Landing during the same flood, and when diversion is occurring, was well illustrated during the high water of 1929. In this year the deposition of silt below the point of

[7] "Improvement of Lower Mississippi River for Flood Control and Navigation", Vol. 1, p. 104.

[8] *Loc. cit.*, Vol. 1, Table 6.

diversion restricted the cross-sectional area to the extent of holding it to the same area during the last 5-ft rise in flood height, as follows:

Gauge height, in feet	Area, in square feet
47.4	234 000
48.8	234 000
49.8	236 000
51.1	230 000
52.2	230 000
52.4	232 000

From the foregoing the deposition of the silt during the same flood is shown to act as a natural automatic valve to hold the cross-section to an area of about bank-full and against a rise in gauge, thus producing a steeper slope during the same flood. This silt is removed during the same flood or intervening waters. The gain in cross-sectional area at this station during the same flood, and when diversion from the Mississippi River is prevented by higher stages in the Red River, is shown by the 1932 flood in the Red River. During this flood a portion of Red River water was discharged into the Mississippi River and increased the volume carried by it. Discharge cross-sections during this year at Red River Landing show:

Gauge, in feet	Area, in square feet
46.6	245 000
48.8	260 000
50.9	267 000
52.1	271 000

The cross-sectional area of 271 000 sq ft at the peak gauge of 52.1 during 1932, as against a cross-sectional area of 232 000 sq ft at a peak gauge of 52.4 ft during 1929, shows a loss of 39 000 sq ft, or 17% of area, caused by diversion during the same flood at this gauge station.

The silting changes of these two floods were only local and did not materially affect the gauge for the same volume of water. The 1932 flood, with diversion eliminated, shows the power of the river to clean the channel and if given the chance, eventually to re-open the entire reach to its old cross-sectional area and produce lower stages for the same volume.

The changes at Red River Landing due to unrestricted diversion are illustrated at the two overbank diversions below New Orleans at Pointe a la Hache and Caernarvon, during the flood of 1927. Discharge observations were taken immediately above and below these diversions. At Pointe a la Hache, 57 miles below New Orleans, 10.5 miles of the Mississippi River levee was removed during 1924 in an effort to reduce flood heights at New Orleans. The 1927 flood discharge observations at Pointe a la Hache show that the velocity above the diversion increased over that below the diversion as the discharge through the levee gap increased. There was a maximum difference in velocity of 1.33 ft per sec, with the maximum diversion of 301 845 cu ft per sec. After diversion the cross-sectional area at the upper station was found to have increased 8 125 sq ft and that at the lower station

had decreased 8 169 sq ft. The increase in area of the upper section was caused by an increase in velocity induced by the suction through the levee gap, thereby increasing the sediment-carrying capacity of the river, adding to its load by scour from the channel within the zone of influence of the diversion. Contrarily, the loss in area at the lower section was due to the decrease in velocity, thus causing deposition of sediment in the channel.

The induced draft through the levee gap at Pointe a la Hache during the 1927 flood did not reach far enough up stream to relieve New Orleans, making it necessary for a further levee breach nearer that city. An artificial crevasse was then made at Caernarvon twenty miles below New Orleans. The greatest difference in velocity between the upper and lower stations at the Caernarvon diversion was 1.13 cu ft per sec for the maximum diversion of 326 446 cu ft per sec. The discharge measurements were not carried through the entire period of diversion and, therefore, cross-sectional comparisons are not possible. These effects from the drop in velocity, and the gain and loss in channel area caused by overbank diversion are noticeable at Red River Landing in the greater channel depth immediately above, and the shallow channel depth immediately below, Old River.

As the peak-flood gauge increased 7.3 ft caused by higher flood stages, due to increased height and strength of levees, larger volumes of water were diverted from the Mississippi River and forced down the Atchafalaya. The higher the gauge at Red River Landing the larger the volume diverted into the Atchafalaya. The Mississippi River, below the point of diversion, adjusted itself with silt deposition, progressively, with increased flood heights and consequent increase in diversion. The loss in mean depth for bank-full stage from 1882 to 1924 was 9.6 ft. Under this unrestricted diversion, it is apparent that the Mississippi River is adjusting itself to the loss in volume. Diversion into the Atchafalaya River is dividing the Mississippi River, and this results in a velocity drop below the point of diversion as compared with that above it. The larger the volume of water diverted the greater the velocity difference. This velocity change is causing a deposition of silt which forms a dam below the point of diversion, thereby adjusting the river to a new slope. This change in slope varies with the volume subtracted. The "efficiency" of the stream down stream from this point is diminished, and, the river at this point is almost continuously subject to diversion and only infrequently is its volume augmented by waters from Red River. The preponderance of influence is due to diversion occurring at all stages from the Mississippi into the Atchafalaya; therefore, there is no opportunity for the sediment-formed dam at Red River Landing to be cleaned out. As the volume of the main river is reduced by increased diversion through the Atchafalaya, the Mississippi River adjusts itself to an increased slope by the deposition of silt below, thus causing (since 1882) a rise in the gauge of 6.9 ft to discharge the same volume of water.

The adjustment at Red River Landing conforms to the hydraulic theory that "whenever silt-bearing streams flow through alluvual deposits, other conditions being the same, the slope is least where the volume is greatest, and,

conversely, the slope is found to be invariably increased as the volume is diminished."[9]

CARROLLTON, LA. (NEW ORLEANS), GAUGE STATION

All available discharges used were measured with current meters and those varying more than 25 000 cu ft per sec from a volume of 1 100 000 cu ft per sec were rejected. Readings were adjusted to the basis of 1 100 000 cu ft per sec, using 60 000 cu ft per sec per ft of gauge, as shown by the 1929 discharge. Only actual observed discharges during 1929 were used.

The record of three crevasses that have occurred within 50 miles up stream and down stream from the Carrollton gauge is given in Table 3 (*c*). No records are available of crevasses that occurred in 1883. Between February 13 and March 26, 1890, ten small crevasses occurred, between Miles 973 and 1 000, down stream from Cairo. These breaks varied, in widths, from 15 to 540 ft and were too small to affect the gauge readings. Furthermore, gauges adjusted for 1 100 000 cu ft per sec before and after the Nita crevasse, which occurred on March 14, 1890 (see Table 3 (*c*)), do not show any influence. The Ames crevasse on March 16, 1891 (see Table 3 (*c*)), occurred after the dates of the gauge readings under consideration. Between May 3 and June 13, 1892, seventeen crevasses are listed. All these breaks were small and closed within a few days of their occurrence, except Sarpy crevasse, 939 miles from Cairo (see Table 3 (*c*)). A comparison of the adjusted gauge readings for this year with those of 1893 (which were not affected by crevasses) shows no influence.

Referring to Table 2 (*e*), the same volume of water in the two different floods, 49 yr apart, was passing the gauge at heights only 1 ft apart, which shows that the channel had not silted appreciably in that time. The greatest variation for discharging the same volume of water in a single year, occurred in 1913. The difference of 2.6 ft between the highest and lowest gauges of this year, adjusted for a discharge of 1 100 000 cu ft per sec, reflects the effect of "efficiency" during the same period. For the 49-yr period, the greatest variation on the gauge for discharging the same volume of water, is 5.3 ft (Table 2 (*e*)). This large variation is due in part to "efficiency" influence, and in part to conditions created by unrestricted diversion at Red River Landing.

SUMMARY: CARROLLTON, LA. (NEW ORLEANS), GAUGE STATION

The study at the Carrollton gauge station is based on a discharge of 1 100 000 cu ft per sec, details of which are recorded in Table 1 (*d*). This volume of water is close to bank-full stage and recorded gauge readings are not affected by crevasses or levee changes.

The variation of 1 ft between the lowest gauge of 1932 and the highest gauge of 1883 (see Table 2 (*e*)) shows that at one time during these two floods, 49 yr apart, the same volume of water was passing within a foot on the gauge. This variation is within the range of "efficiency" influence

[9] Letter from the late James B. Eads, F. Am. Soc. C. E., to Mississippi River Comm. dated April 12, 1882.

and the channel has not silted. The greatest variation for discharging the same volume of water occurred in 1913. The difference of 2.6 ft between the highest and lowest gauge of this year for discharging 1 100 000 cu ft per sec, is due in part to "efficiency" influence. This variation of 2.6 ft is greater than can normally be expected due to efficiency influence, as the conditions set up during the flood of 1913 at Red River Landing caused an abnormal gauge fluctuation at Carrollton. Prior and subsequent to the flood of 1913 the greatest gauge variation during any one year for discharging this same volume of water is 1.5 ft. For the 49-yr period the greatest range in gauge for discharging this same volume of water is 5.3 ft. The increase in gauge for discharging the same volume of water at this station

TABLE 6.—COMPARISON OF CROSS-SECTION ELEMENTS, STAGE BANK-FULL: ATCHAFALAYA RIVER; SURVEYS OF 1904–05, 1916–17, 1931, AND 1932

Reach	STATIONS, IN MILES*		WIDTHS, IN FEET				Areas, in Square Feet	
	From: (1)	To: (2)	1904–05 (3)	1916–17 (4)	1931† (5)	1932† (6)	1904–05 (7)	1916–17 (8)
0	0.0	3.3	1 147	1 173	1 395	1 420	58 456	67 937
1	3.3	8.1	1 177	1 143	1 277	1 304	52 792	58 717
2	8.1	13.1	1 135	1 251	1 423	1 469	58 018	65 483
3	13.1	18.1	1 116	1 152	1 326	1 370	55 966	63 556
4	18.1	23.0	1 017	1 044	1 204	1 216	54 588	57 382
5	23.0	28.0	1 044	1 119	1 200	1 210	58 529	68 368
6	28.0	32.9	1 044	1 086	1 163	1 170	55 222	58 803
7	32.9	37.9	920	976	1 052	1 062	46 479	53 859
8	37.9	42.9	719	834	980	987	31 769	40 553
9	42.9	47.8	579	872	1 148	1 158	19 418	41 126
10	47.8	52.8	527	751	1 094	1 097	19 121	39 749
11	52.8	57.9	434	572	870	887	15 107	27 041
12	57.9	63.0	367	390	596	592	12 604	15 729
13	63.0	68.0	494	509	640	638	15 184	16 819
Total			11 720	12 874	15 368	15 580	554 253	675 132
Average			837	920	1 098	1 113	39 590	48 224

Reach	AREAS, IN SQUARE FEET (*Continued*)		MAXIMUM DEPTHS, IN FEET				Stage elevation, in feet, above mean Gulf level
	1931† (9)	1932† (10)	1904–05 (11)	1916–17 (12)	1931 (13)	1932 (14)	(15)
0	64 724	79 056	88.2	91.5	77.9	100.5	47.72
1	59 449	70 462	69.9	78.8	76.3	88.3	45.71
2	67 498	74 362	79.7	83.5	83.4	91.8	43.65
3	66 572	72 542	80.7	86.0	87.4	92.5	41.52
4	70 812	71 829	77.4	84.5	94.2	94.8	38.79
5	73 063	74 019	93.0	98.1	97.3	99.5	35.59
6	57 701	57 519	87.6	87.6	83.5	82.5	32.14
7	58 544	59 573	81.4	86.2	85.5	84.1	29.07
8	53 714	54 737	68.4	78.2	84.5	85.7	26.59
9	57 657	58 210	54.6	80.6	81.4	83.3	24.30
10	57 438	58 830	58.5	80.4	88.8	90.7	21.23
11	43 263	45 447	58.3	70.4	83.3	85.5	17.92
12	24 804	26 140	56.2	59.2	69.0	69.7	15.38
13	22 665	23 716	51.8	50.3	57.3	58.4	13.29
Total	777 804	826 442	1005.7	1115.3	1149.8	1207.3	432.90
Average	55 557	59 032	71.8	79.7	82.1	86.2	30.92

* Station 0 is at Barbres Landing, La. (see Fig. 2.).
† Sections normal to stream computed where necessary.

is between 3 and 4 ft. This may be caused by conditions set up by unrestricted diversion at Red River Landing. Definite proof is not available at this time, however, but it is thought that future studies will throw additional light on this subject.

THE DISCHARGE OF THE ATCHAFALAYA RIVER

The cross-sectional elements of the Atchafalaya River for the upper 68 miles, as taken by the Mississippi River Commission for various years (Table 6), show that this stream has increased an average of 47% in area at bank-full stage from 1904 to 1932. (Levees have been constructed along the first 51 miles of this river.) In order to determine the effect, if any, that this large increase in cross-sectional area has had on the discharge capacity of the stream, it is necessary to consider the passage of the same volume of water through the various flood-years. To obtain a sufficient number of readings for consideration, a volume of 300 000 cu ft per sec is used as reported at the Simmesport Discharge Observation Station (see Table 1(*e*) and Table 2 (*f*)). As in the case of gauge records previously reported in this paper, all discharges in Table 1 (*e*) were measured with current meters and those varying more than 15 000 cu ft per sec from a volume of 300 000 cu ft per sec were rejected. At this gauge the readings were adjusted to a basis of 300 000 cu ft per sec, using 13 000 cu ft per sec per ft of gauge, as shown by the discharge rating curve issued by the Corps of Engineers, U. S. Army, for this station. Readings from 1890 to 1893, which are published with the zero of the gauge at 3.88 ft above mean Gulf level were corrected to a gauge with its zero 5.79 ft above mean Gulf level, thus referring all readings in Table 1 (*e*) to the same datum plane. A record of the more important crevasses along the Atchafalaya River is presented in Table 7.

TABLE 7.—RECORD OF IMPORTANT CREVASSES

Name (1)	Bank (2)	Date (3)	Width, in feet (4)	Name (1)	Bank (2)	Date (3)	Width, in feet (4)
Barbres	Left	April 21, 1890	215	Burton	Right	April 22, 1890	1 600
Cottage Point	Left	April 21, 1890	600	Churchville	Right	April 22, 1890	500
Yoist	Left	April 21, 1890	1 800	Bayou Marine	Left	April 15, 1890	550
School	Left	April 21, 1890	110	Pouncey	Left	1891	.....
Jacoba	Left	April 21, 1890	150	Deer Range	Right	1891	.....
Callahan	Left	April 21, 1890	180	Nelson-Eddy	.....	1893	.....
Mock	Left	April 21, 1890	480	Holloway	Left	1903	.....
Smith and Taylor	Left	April 21, 1890	50	Alto	Right	May 19, 1912	2 450
Harmanson	Right	April 8, 1890	300	Atkins	Right	May 17, 1912	.....
Norwood	Right	April 8, 1890	150	McCracken	Left	April 16, 1912	5 200
Yellow Bayou	Right	April 8, 1890	200	Coville	Left	April 25, 1913	400
Cason	Right	April 15, 1890	300	McCrea	Left	May 24, 1927	.....
Gordon	Right	April 18, 1890	400	Melville	Right	May 27, 1927	.....

There is wide variation in the "efficiency" of the Atchafalaya River. The difference of 6.1 ft between the 1920 and the 1928 readings, eight years apart (see Table 2 (*f*)), shows the influence on "efficiency" by the sediment deposited in the stream as a result of diversion. The flood of 1927 formed crevasses in the Atchafalaya River levees, flushing the river channel sediment at different

points and allowing efficient handling of the 1928 water at low gauges. The flood in the Red River during 1932 established the highest gauge of record at Alexandria, La. This high stage of the Red River prevented the diversion of the Mississippi into the Atchafalaya and cleaned the Atchafalaya River to greater depths than recorded by surveys since 1880–1881. This flushing increased the Atchafalaya "efficiency," allowing the early part of the 1933 water to pass at low stages.

The gauge variation of 1.7 ft during the 43-yr period, 1890 to 1933 (see Table 2 (*f*)), develops the fact that no material change occurred in the discharge capacity of the stream, even in the face of a 47% increase in cross-sectional area. The velocities in Table 1 (*e*) show that as the river expanded in cross-sectional area there was a reduction in velocity from about 5 ft to less than 4.5 ft per sec caused by slope adjustment of the stream. The head is fixed by the water elevations in the Mississippi River at Red River Landing, and, therefore, the slope is reduced because of the increased silt deposits below the lower end of the levees. Proof of this silting is shown[10] in Table 8. Melville is at the lower end of Reach *B*; and the lower end of Reach *E* is approximately at the end of the east levee. No change in gauge for the discharge of the same volume of water has occurred since 1890; therefore, the increase in cross-sectional area has kept pace and has offset the slope reduction. The theory that with an increase in the volume of a silt-bearing stream the slope will diminish, is borne out by the actions of the Atchafalaya River.

TABLE 8.—ATCHAFALAYA RIVER: CHANGES IN AVERAGE BANK ELEVATION (IN FEET), AS SHOWN BY SURVEYS

Reach	Station, in miles	1880–81 to 1904–05	1904–05 to 1916–17	1916–17 to 1931
		(24 yr)	(12 yr)	(15 yr)
A..............	0–13.7	+1.0	−0.5	+0.8
B..............	13.7–29.8	+3.1	+1.0	+1.7
C..............	29.8–36.9	+3.0	+4.3	+4.0
D..............	36.9–43.0	+1.8	+6.3	+1.7
E..............	43.0–52.9	.....	+5.0	+3.3
F..............	52.9–63.7	.....	+2.5	+5.7
G..............	63.7–66.9	.....	+1.6	+4.8

SIMMESPORT, LA., GAUGE

In 1890, the east levee on the Atchafalaya River extended to 26 miles below Simmesport (see Fig. 2), the west levee extending to 11 miles below. Levee extensions since 1890 have not affected this gauge for the same volume of discharge. For the various flood-years when the high gauge of 50 ft was reached in the Mississippi at Red River Landing, the record of discharge in the Atchafalaya River at Simmesport for corresponding dates is given in Table 9. The discharges vary with the silted condition of the channel and are not upset or influenced by crevasses.

[10] "The Improvement of the Lower Mississippi River for Flood Control and Navigation", May, 1932, prepared under the direction of the President of the Mississippi River Commission, Vol. 1, p. 126.

The Atchafalaya discharge of 376 000 cu ft per sec during 1932 at a gauge of 49.5 ft at Red River Landing (see Table 9, Item No. 7) is within reasonable

TABLE 9.—DISCHARGE IN THE ATCHAFALAYA RIVER

Item No.	Date	Mississippi River gauge, at Red River Landing, in feet	Discharge, Atchafalaya River, at Simmesport, in cubic feet per second
1................	May 10, 1897.....	50.0	390 000
2................	April 8, 1903.....	50.0	388 000
3................	April 20, 1912.....	50.1	331 000
4................	May 2, 1913.....	50.0	390 000
5................	May 4, 1920.....	49.6	306 000
6................	May 2, 1929.....	49.9	346 000
7................	February 17, 1932.	49.5	376 000

limits of the 390 000 cu ft per sec discharged during 1897 at a gauge of 50.0 ft at Red River Landing. (The year, 1897, is the earliest on record in which a gauge of 50.0 ft was reached at Red River Landing.) This further substantiates the flattening of the slope in the Atchafalaya River; it offsets the 47% increase in cross-sectional area; and the volume of water diverted into the Atchafalaya is dependent upon water elevations in the Mississippi River. The increase in discharge of the Atchafalaya River due to the increase of 6.9 ft in the Mississippi for discharging the volume of 1 100 000 cu ft per sec is shown in Table 10.

TABLE 10.—INCREASE IN THE DISCHARGE OF THE ATCHAFALAYA RIVER

Period	Dates	Mississippi River, Red River Landing		Atchafalaya River, Simmesport discharge, in cubic feet per second
		Gauge height, in feet	Discharge, in cubic feet per second	
1............	1882–1885	41.3	1 108 600	184 421
3............	1913–1932	48.2	1 105 000	313 866
Increase......	....	6.9 ft	....	129 445

MELVILLE AND KROTZ SPRINGS GAUGE STATIONS

Discharge observations are not made in the Atchafalaya River, at Melville and Krotz Springs, La. (see Fig. 2). To study the action of the river at these two points gauge heights are listed in Table 11 for the day following the approximate discharge of 300 000 cu ft per sec at Simmesport. The dates for the gauges at Melville and Krotz Springs are one day later than those for the gauges at Simmesport to correspond with a discharge of 300 000 cu ft per sec. The zero for 1933 values on the Melville gauge is 0.2 ft above mean Gulf level. To convert the gauge readings for previous years to the same datum plane as 1933, subtract 0.3 ft from the readings in 1890 and 1892, and add 0.2 ft to the readings in 1907 and 1929. The gauge readings in Table 11 are not adjusted to an even 300 000 cu ft per sec.

At Melville the gauges from 1907 to 1933 do not show any noticeable changes other than those due to "efficiency" influence. The average of the gauge readings prior to 1907 is 5.2 ft lower than the average of those after

that year. This increase in gauge between 1892 and 1907 is due to levee confinement. In 1893 the west bank of the Atchafalaya River had a continuous levee line extending to Melville and the east levee line ended 8 miles up stream from this point. Between 1893 and 1907 the west levee was extended to 12 miles below Melville and the east levee extended to 13 miles below that point. This confinement caused the increase in gauge heights of approximately 5 ft at Melville.

TABLE 11.—GAUGE READINGS AT MELVILLE, LA., AND KROTZ SPRINGS, LA. (SEE FIG. 2)

Date	Gauge height, in feet, Melville, La.	Date	Gauge Height, in Feet		Date	Gauge Height, in Feet	
			Melville, La.	Krotz Springs, La.		Melville, La.	Krotz Springs, La.
April 8, 1890....	33.7	June 17, 1908..	39.8	....	April 22, 1933..	36.4	52.4
March 26, 1891..	33.3	April 14, 1912..	39.8	53.7	April 23, 1933..	36.6	52.5
April 18, 1891...	33.4	April 25, 1920..	41.3	54.9	April 25, 1933..	37.0	52.7
May 21, 1892....	33.4	April 28, 1920..	41.6	55.0	April 27, 1933..	37.3	52.9
May 29, 1892....	35.5	May 4, 1920...	41.9	55.2	April 28, 1933..	37.4	53.0
February 15, 1907	37.6	April 14, 1923..	39.8	....	May 2, 1933....	37.8	53.4
February 17, 1907	37.8	July 22, 1928..	37.7	51.9	May 4, 1933...	38.1	53.6
April 7, 1908....	41.1	April 7, 1929..	39.0	53.0	May 6, 1933....	38.3	53.7
May 5, 1908....	38.3	April 11, 1929..	39.6	53.3	May 14, 1933..	38.2	53.7
May 7, 1908....	38.5	....	......	....	....	......	....

The gauge at Krotz Springs from 1912 (the earliest gauge reading available) to 1933 does not show any noticeable change other than that due to "efficiency" influnce. In 1912 the east levee ended 3.7 miles below this station and has since been extended to 4 miles below it (see Fig. 2). In 1908 the west levee ended 1 mile below this station and has since been extended to 9 miles below it. These levee extensions have not affected this gauge for the same volume of discharge.

HISTORY OF THE ATCHAFALAYA RIVER BASIN[11]

In 1804, the channel of the Atchafalaya River was choked with rafts of driftwood extending from bank to bank, beginning on the north at a point 30 miles from the Mississippi River and extending down the Atchafalaya about 20 miles. They clogged the river channel completely, making navigation impossible. Trees were growing around these rafts, the largest being about 10 in. in diameter. Driveways crossed them at different places. These rafts rose and fell with the water and, apparently, they were effective in checking any natural opening of this channel between Old River and the Gulf.

About 1840 the State of Louisiana, in order to open a navigable channel, began cutting channels through the rafts with a view to their complete removal. This work continued to the beginning of the Civil War period. Immediately upon the removal of the rafts the channel enlarged rapidly at bends in the river and on both sides of the narrow straight reaches. Lands in the Atchafalaya Basin which had been exempt from overflow were flooded annually until, in 1881, the Upper Atchafalaya Basin "had gone back to Nature."

[11] Comp. from Repts. of the Mississippi River Commission.

Along the river were to be seen ruined dwellings and sugar houses, buried fence lines, broken levees, hardwood timber killed by standing water, and cypress swamp growth started. Navigation, however, was excellent in 1881. The change in the characteristics of the river at its head between 1851 and 1879, is illustrated by reference to Table 12.

TABLE 12.—CHANGES IN THE CHARACTERISTICS OF ATCHAFALAYA RIVER

Year	Cross-section area of channel, in square feet	DIMENSIONS OF CROSS-SECTION AT HIGH WATER, IN FEET	
		Width	Depth
1851	24 400	730	52
1859	28 700	830	63
1874	39 160	891	114
1879	52 100	940	130

In 1881, the channel of the Atchafalaya River had enlarged to a point at which it had a discharge nearly equal to that of the Red River and afforded that stream the line of least resistance for flow to the sea. In 1881, the elevation of the water surface at Red River Landing was almost constantly above that at the head of the Atchafalaya, being 7.3 ft above the latter point on October 13, 1891. At this time there was a marked tendency to increase the channel from the Mississippi to the head of the Atchafalaya. Quoting from the report of the Commission:[12]

"No decrease in flood heights in the Mississippi River has been observed, although a diversion of one-sixth of the Mississippi River discharge has taken place. However, there has been a decrease of flood heights on the Atchafalaya."

The report of the Mississippi River Commission of 1881 contained a recommendation that a sill dam be built across the mouth of Old River between Turnbull's Island and the Mississippi, and, also, that a study be made of the proposal to divorce completely the Mississippi River from the Red and Atchafalaya drainage. During 1882 actual work at the head of the Atchafalaya was confined to dredging bars at the mouth of the Red River and the head of the Atchafalaya, creating a 16-ft channel from the Red River through Old River and a 6-ft channel connecting the Red with the head of the Atchafalaya River, thus making navigation conditions very satisfactory. In 1884 the Commission reported[13] that the formulation of a comprehensive plan for the treatment of the Atchafalaya situation had been deferred from time to time on account of the necessity for further observation, experience, and study. It was prepared to present a plan which was stated to be the best that could be devised in point of practicability, safety and economy; a plan that should include "the prevention of the diversion of the Mississippi into the Atchafalaya Basin and the closure of any depleting outlet in that point, either now existing or likely to be induced by the changes reasonably to be anticipated and also the preservation of navigation of the Red and the Atchafalaya Rivers."

[12] Progress Rept. of the Mississippi River Comm., 1881, p. 13.
[13] Rept. of the Mississippi River Comm., 1884, p. 2554.

At that time Major Amos Stickney, Corps of Engineers, U. S. Army, was in charge of the lower district, and he recommended the construction of a series of brush dams to be placed in the Atchafalaya River, the upper one to be below the Bayou des Glaizes and the others at intervals not exceeding $\frac{1}{4}$ mile. These dams were to be built to a point just below low water; they were planned not to interfere with navigation, and to regulate the discharge of the Atchafalaya so that it would take care of the flood discharge of the Red River. These dams were to be supplemented by levees to prevent the overflow into the Atchafalaya Basin of water from the Mississippi. At this time a dam was also recommended across Old River between the Atchafalaya and the mouth of the Red near Turnbull's Island. This was in order to force the Red River down the Atchafalaya. Apparently, no action was taken by Congress on this recommendation and the report of 1885 again recommends the construction of these sill dams in the Atchafalaya as soon as the water stage permits and money is available. Quoting the report:[14]

"The work to be done would have for its object the gradual construction and perhaps finally the complete closure of the Atchafalaya as an outlet of the Mississippi. Whether the treatment should be carried further than this, to the extent of making the Red a tributary of the Mississippi by the construction of a high dam across the head of the Atchafalaya River and its basin, thus cutting the latter off from navigation of the Red and Mississippi, is a consideration of great moment. Determination can be deferred. The Commission is distinctly committed to the idea of closing all outlets as part of the plan for the improvement of the Mississippi River, both low-water outlets and breaks in the levees, and has consistently opposed the fallacy known as the outlet system because it stands in sharp antagonism to all the fundamental laws of hydraulics. When the Atchafalaya shall cease to be an outlet the discharge of the Mississippi below the mouth of the Red will be decreased from 40 to 50 per cent. by the addition of volume now substracted by the outlet."

The report of the Commission of 1886 states "that no instructions have been received for carrying out the foregoing plan." During 1888 construction on the sill dams in the Atchafalaya below Simmesport, La., was started, in line with recommendations previously made by the Commission. The first sill dam was located 500 ft below the mouth of the Bayou des Glaizes. This report contains complete descriptions and analyses of a number of plans for handling the Red River-Atchafalaya Basin problem. These data appear to be segregated about as follows: (1) Major Stickney's plan (six submerged sill dams); (2) Eads' plan (to cut off the Atchafalaya River with a high dam at the head); (3) canal and lock plan (cut the canal, Mississippi to Atchafalya Rivers, with locks therein); (4) Mississippi River Commission plan (includes Plan (1) and also a plan to shut off the connection between the mouth of the Red and the Atchafalaya Rivers with a sill dam on the west side of Turnbull's Island); and, (5) variation of Plan (4) (includes a long jetty from the lower point of Turnbull's Island extending into the Mississippi to prevent direct flow from the latter into Old River).

[14] Annual Rept. of Mississippi River Comm., 1885, p. 2872; Appendix WW of the Annual Rept. of the Chf. of Engrs., 1885.

The report of 1889 indicates that Plan (5) had been adopted for handling the situation. During the year the upper sill dam in the Atchafalaya was practically completed and the second sill dam had been commenced. This second dam is located 1 750 ft down stream from Sill Dam No. 1. The report of 1890 shows the completion of the second sill dam on August 27. During the months of September, October, and November, 1890, the sill or lower course of a sill dam was constructed across Old River, $\frac{3}{4}$ mile down stream from Sugar House Chute. The reports of 1891 and 1892 show progress on the construction of this sill dam, which was practically completed in 1892.

To summarize, then, from 1804 to 1840, the Atchafalaya was completely blocked by rafts and received very little flood water from the Mississippi River. In 1852, according to an estimate by Charles Ellet, a Civil Engineer employed by the Government, it was handling one-twelfth of the Mississippi discharge. In 1882, Mr. Eads estimated that it was carrying approximately one-sixth of the Mississippi River outflow. At present (1935) the Atchafalaya is discharging, during flood periods, a levee-confined volume of about 500 000 cu ft per sec, or one-third of the discharge of the Mississippi River below Red River Landing. The sill dams, the construction of which has been mentioned previously, were designed and built in order to restrict the discharge of the Atchafalaya to about 200 000 cu ft per sec, which was estimated to be the discharge of the Red River while in flood at that time (1882). These dams did not restrict the Atchafalaya to a capacity of 200 000 cu ft per sec, due to later increased flood heights caused by the construction of higher and stronger levees. Both sill dams are alike in design and construction. The base or sill consists of a willow mattress, 3 ft thick, and 304 ft long, up stream and down stream, loaded with rock. Upon this sill were built three courses of willow mattress, 100 ft, 66 ft, and 30 ft in width, respectively, each course being covered with successive layers of hard clay and gravel. The up-stream edge of the dam is 20 ft below the upper edge of the sill. The entire structure was given a heavy covering of rock.

Conclusions

Except where indicated the gauge and discharge readings in this paper are not affected by crevasses, cut-offs, levee extensions, or enlargements. At the gauge stations along the Mississippi River, at Columbus, Ky., Arkansas City, Ark., and Helena, Ark., the flow is not subjected to unrestricted diversion and the same volume of water is now passing at the same elevations as in earlier years. At these points the river is not silting, but a variation in gauge due to "efficiency" influence of approximately 3 ft can be expected for the same volume of water. Overbank diversion occurred at the Arkansas City gauge station prior to 1921 through the gap at Cypress Creek, a short distance above, without affecting the gauge in passing the same volume of water.

At Red River Landing, the Mississippi River is divided into two levee-confined outlets to the Gulf, permitting diversion from the Mississippi into the Atchafalaya River at all stages, except when higher stages occur in the Red River. The higher stage in Red River is infrequent compared to diversion from the Mississippi. This division of the river causes a drop in velocity

below the point of diversion over the velocity of that above, the difference being greater as the volume diverted increases. This checking of the current in flood time causes a deposition of some part of the silt load, and it is in this manner that Nature provides the means of the river's adjustment. The greater the diversion, the greater will be the difference between the velocity above and below, and, consequently, the greater the deposition of silt below the point of diversion. The levees on the main river were raised and strengthened, thus causing levee confinement to larger volumes of water. Increased flood heights resulted and raised the average flood peaks at Red River Landing 7.3 ft between the periods 1867 to 1886 and 1912 to 1933. This peak-flood increase caused larger volumes of water to be diverted from the Mississippi through the Atchafalaya. The Mississippi River below the point of diversion, adjusted itself progressively with increased flood heights and consequent increase in diversion by the river channel being partly closed by a silt-deposited dam. This dam increased the river slope for the loss in volume diverted and raised the gauge 6.9 ft to discharge the same volume of water since 1882. The close relationship between the increase in peak-flood heights and the increase in gauge for discharging the same volume of water is apparent as only a difference of 0.4 ft exists.

At Carrollton the gauge has been raised between 3 and 4 ft since 1882 in passing the same volume of water. Definite proof as to the cause of this increase is not available. The head of the Atchafalaya is fixed by the water elevation in the Mississippi River at Red River Landing. Consequently, the slope has been adjusted by silt deposited below the lower limit of the levee influence. Although the velocities have decreased in the Atchafalaya with the diminished slope the increase in bank-full cross-sectional area of 47% since 1904 has kept pace and offset the effect of slope reduction, so that the same volume of water at this time is passing the same gauge height as it was in 1890. This balancing of the lessened slope by the enlargement in cross-sectional area has made it possible to increase the volume of discharge through the Atchafalaya only by increased flood heights in the Mississippi River. This is definitely shown by the discharges of the Atchafalaya River at Simmesport for a high gauge of 50 in the Mississippi River (see Table 9, Items Nos. 1, 2, 4, and 7).

The distance from Red River to the sea *via* the Mississippi River is 310 miles and *via* the Atchafalaya River, 125 miles. The shorter distance and steeper slope afforded by the Atchafalaya appears to be a logical course for the Mississippi River to follow. This condition existed for centuries before Man made levees, giving every opportunity for the Mississippi River to take advantage of this short route to the sea if Nature so desired. Through the manner in which the river treats the silt load the shorter route to the sea is blocked and, in addition, makes it necessary to increase the head to increase the discharge. The discharge of the Mississippi River at Red River Landing is 35 000 cu ft per sec per ft of gauge. The increase of 6.9 ft in gauge since 1882 at Red River Landing for a discharge of 1 100 000 cu ft per sec means that the Mississippi River has lost 240 000 cu ft per sec in discharge capacity. At the same time, the discharge of the Atchafalaya River at Simmesport

increased 129 000 cu ft per sec, due to this additional head of 6.9 ft. The combined loss in discharge capacity of 111 000 cu ft per sec results from unrestricted diversion.

The Mississippi River has conformed itself to the hydraulic theory and increased its slope as the volume has been diminished through diversion. Likewise, the Atchafalaya River has conformed itself to the hydraulic theory and flattened its slope as its volume increased through diversion. It is obvious that the vicious cycle of conditions set up through unrestricted diversion from the Mississippi into the Atchafalaya results in a large loss in "efficiency."

The treatment of the silt load by a silt-bearing stream is its powerful agency of adjustment to changed conditions. The detrimental effects resulting from this power to adjust itself to changed conditions is reflected clearly in the loss in discharge capacity of the Mississippi River at Red River Landing due to unrestricted diversion at all stages. By eliminating diversion below bank-full stage a major portion of this power of adjustment is overcome as it will disturb to a minor extent the greater portion of the silt load that is handled along the bed of the channel. Furthermore, the agency is provided for waters below bank-full stage, acting for 365 days to clean out any deposition of silt left after the short duration of over-bank diversion. This method of diversion will practically restore the channel to its original discharge capacity and so maintain it. The slight loss in cross-sectional area during overbank diversion at Pointe a la Hache and the fact that overbank diversion occurred at Arkansas City for a number of years did not reduce the discharge capacity of the river, substantiate the conclusion that overbank diversion will not cause the ill effects suffered from diversion at below bank-full stages.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

STABLE CHANNELS IN ERODIBLE MATERIAL

By E. W. Lane,[1] M. Am. Soc. C. E.

Synopsis

The design of the All-American Canal, which will divert 15 000 cu ft per sec from the Colorado River, required a thorough study of stable channel shapes. Data from various sources were conflicting and unsuitable for the unusual conditions on this canal. These data were analyzed and conclusions were drawn regarding the various factors controlling stable channel shapes, and the relation between them.

Introduction

For a canal to be stable the banks must not slough or slide, and the bottom and sides must neither silt nor scour. To establish these conditions the engineer must consider a number of factors.

The problem of controlling sloughing or sliding is not treated in detail in this paper, since stable slopes for the various soils are comparatively well known. To prevent scouring or the accumulation of silt in the bed, it is necessary that the velocity along the bed is sufficient to move all the material brought into the canal, and yet not be so high as to cause the subgrade of the canal to scour. Flowing water will not attack the subgrade unless its velocity is more than sufficient to move the material brought into the canal. The excess of velocity over that required to move this material, which will attack the canal subgrade, depends upon the material of which the subgrade is composed.

In order that the banks may neither silt nor scour, the velocities along them must be sufficient to prevent deposition but not sufficient to cause the material of which they are composed to scour. From a practical standpoint a slight degree of silting on the sides is not especially detrimental, so that the important requirement is the prevention of scour and excessive deposition. The maximum allowable velocity along the banks depends upon the material of which they are composed. The material on the sides is also acted upon by the force of gravity, which assists the water in tending to cause motion. The sides, therefore, will scour under velocities less than would be permissible along the bottom.

Note.—Discussion on this paper will be closed in February, 1936, *Proceedings*.

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The ratio between the velocity acting on the sides and that on the bottom depends upon the ratio of the bed width of the canal to the depth. The greater this latter ratio the greater will be the ratio of the velocity acting on the bottom to that acting on the sides. The bed width-depth ratio required for a stable channel is that which will bring the proper ratio of velocity acting on the bottom to that acting on the sides. Conditions that require high velocities acting on the bottom, as compared with those that may be permitted to act on the sides, require high ratios of bed width to depth. For example, canals that carry a heavy bed load in friable material require high velocities on the bed to move the load and low velocities along the banks to prevent cutting them; in other words, such channels require a high ratio of bottom velocity to side velocity, and, therefore, a high bed width-depth ratio. Canals with small bed loads in friable material do not require such high velocities along the bottom to transport the bed load and, therefore, the ratio of this lower velocity, V_B, to the permissible side velocity, V_S, can be smaller. The correct ratios for other conditions can be determined by the application of these principles. This paper records an attempt to outline the major principles that control stable channel shapes and velocities.

The All-American Canal

The All-American Canal is planned to take water from the Lower Colorado River and carry it to the lands lying in the Imperial and Coachella Valleys by a route lying entirely within the United States. The Imperial Valley is now (1935) irrigated from the Colorado River by a publicly owned canal system, of which a large part of the main canal lies within the Republic of Mexico. The difficulties of international administration, and the undesirable silt conditions connected with the existing canal has led to the instigation by the United States Bureau of Reclamation of the new All-American Canal project to be built entirely within the United States, which has been approved by Congress, and is now under construction.

The Colorado River is a very silt-laden stream. It has a discharge varying from 2 500 to 190 000 cu ft per sec and a suspended silt content near the intake of the proposed canal averaging 0.90% by weight, and, at times, reaching 5.40 per cent. The suspended silt is extremely fine and the bed silt averages about 0.10 mm (0.004 in.) in diameter. The river slope is approximately 1.2 ft per mile. The use of this very silty water in the Imperial Valley has led to great difficulty and an expense estimated at approximately $1 400 000 per yr for dredging, canal cleaning, and land leveling.

Before the All-American Canal is finished, it is expected that the Boulder Dam, 303 miles above the proposed head-works, will be completed. This dam will be 725 ft high and will form a reservoir, with a maximum capacity of 30 500 000 acre-ft, in which all the silt brought down to the reservoir will be deposited. Another dam will be built 155 miles above the intake, which will stop practically all the silt coming from above that point. The silt that reaches the intake, therefore, will be only that picked up from the banks and bed of the river below the lower dam, and the small quantity brought in by the tributaries to the river between the lower dam and the intake. An

elaborate desilting works will be built to remove the coarser part of the silt load which will be carried into the canal. Consequently, the silt load in the canal will differ greatly from that now carried, and will create different stable channel shapes. To determine the shape best adapted to the new condition a thorough study of the subject was made, the results of which are described herein.

Notation

The symbols introduced in this paper are defined as follows (the English system of units being used unless otherwise stated):

a = a subscript denoting "average";
d = depth of flow; d_a = average depth;
f = silt factor $= 8\sqrt{D}$;
m = a subscript denoting "mean";
n = exponent in a formula of the Kennedy type;
s = a subscript denoting "near the sides";
A = area of water cross-section;
B = breadth, or width of channel; B_B = width at the bottom; B_m = mean channel width; as a subscript, B, denotes "at or near the bottom";
C = coefficient in a formula of the Kennedy type;
D = diameter of particles;
L = length;
P = wetted perimeter (not including water surface);
Q = flow, or rate of discharge;
R = hydraulic radius $= \frac{A}{P}$;
S = slope $= \frac{\text{fall}}{L}$
V = velocity; average velocity in a section; V_B = velocity near the bottom; V_S = velocity near the sides; and V_0 = critical velocity from the standpoint of silting.

History of Non-Silting Canal Section Studies

Most of the study of the problem of non-silting canal sections has been made by the British engineers in India, in connection with the large irrigation projects of that country. A certain amount has also been done in Egypt, in connection with the irrigation works on the Nile. Thus, far, little has been contributed by the United States. In the last few years, however, a surprising interest in silt problems has developed in this country and it is now (1935) being attacked from many angles by a number of research engineers. From this interest, no doubt, future progress will become considerably more marked.

The first study of non-silting canal sections was made by Mr. R. G. Kennedy (1)[2]. His work is a classic in this field, and has resulted in the saving of millions of dollars in reducing the cost of cleaning irrigation canals in India and elsewhere. Unfortunately, like most outstanding studies, it came to have such prestige that for many years little further progress along this line was made.

[2] For reference to figures in parentheses see "Bibliography."

Kennedy gave the result of measurements of bed widths and "full supply" depths on about twenty-two canals in the Lower Bari Doab Canal System, in which the channels had become stable and several more which had nearly reached this condition. He also gave the "full supply" discharge and the velocity computed from this discharge and the full supply area. From these data he developed a formula of the type:

$$V_0 = C d^n \qquad (1)$$

which expressed, with reasonable accuracy, the relation between the critical mean velocity, V_0, and the depth, d, as indicated by the results of the measurements. For the Lower Bari Doab Canal, C was 0.84 and n was 0.64. Kennedy expected C to vary with the quality and quantity of silt, but thought n would be nearly constant. On Fig. 1 (with reference to Table 1) is shown a line giving the velocities corresponding to the various depths according to Equation (1). The local conditions influencing these observations and Kennedy's conclusions will be given more in detail subsequently.

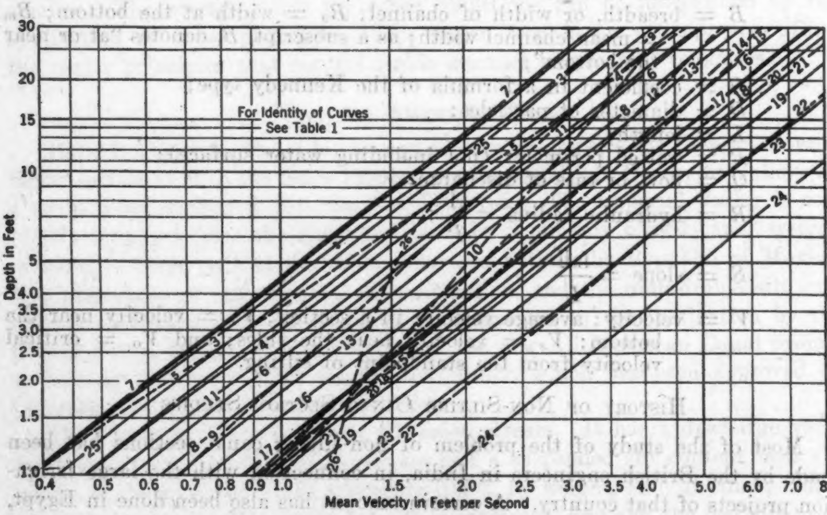

FIG. 1.—CRITICAL VELOCITY FORMULAS FOR NON-SILTING, NON-SCOURING VELOCITIES.

In 1895 Kennedy issued a set of hydraulic diagrams to aid in the design of non-silting channels. In 1904, he gave a rough rule for the relation of width to depth in non-silting canals (5). A second edition of "Hydraulic Diagrams" (6) was issued in 1907, in which Kennedy reprinted the original paper and added an extended discussion to clarify some of the obscure points and to give the results of his experience since the first paper was printed.

Kennedy's work soon became extensively used throughout India; observations were made on the ditches of other irrigation systems and a number of other equations of the same type as those of Kennedy, were developed, suitable to the various local conditions. One of these was for the Godavari Western Delta and the Kistna Western Delta, in Madras (7). In 1913 a set

TABLE 1.—VALUES OF C AND n, EQUATION (1), FOR NON-SILTING, NON-SCOURING VELOCITIES

Curve No. (see Fig. 1)	Factors Applying to Equation (1) (in English Units)		Locality	Authority	Limit *	Reference
	Coefficient C	Exponent n				
1	0.381	0.64	Egypt	Buckley	Lower	Irrig. Dept. of Egypt.
2	0.46	0.64	Egypt	Buckley	Upper	
3	0.39	2/3	Egypt	Molesworth and Yenidunia	Lower	"Irrigation Practice," p. 207.
4	0.475	2/3	Egypt		Upper	
5	0.391	0.727	Egypt	K. D. Ghaleb	....	*Minutes of Proceedings*, Inst.. C. E, p. 260, Vol. 229; also p. 285, Vol. 223.
6	0.56	0.64	Egypt	....	....	U. S. Dept. of Agri., *Technical Bulletin No. 67*, p. 44.
7	0.38	0.64	Mozaffargorih D. Punjab, India.	G. W. Duthy	Lower	*Proceedings*, Punjab Eng. Congress, pp. 44 and 48, 1919.
8	0.63	0.64		G. W. Duthy	Upper	
	0.63	0.64	Sind	F. W. Woods	....	*Engineer*, p. 648, Vol. 143; Parker, "Control of Water," p. 678.
9	0.67	0.55	Godavari Western Delta, Madras.	Kennedy	....	*Minutes of Proceedings*, Inst. C. E., p. 260, Vol. 229; Madras Public Works Dept., Oct. 9, 1912, Dist. 1872.
10	1.01	0.44	Rio Negro, Argentina	R. E. Ballester	....	*Minutes of Proceedings*, Inst. C. E., p. 280, Vol. 223.
11	0.52	0.66	Siam	....	....	U. S. Dept. of Agri., *Technical Bulletin No. 67*, p. 44.
12	0.93	0.52	Madras (Kistna)	Kennedy	....	*Minutes of Proceedings*, Inst. C. E., p. 260, Vol. 229.
13	0.67	0.64	Sutlej, India	F. W. Woods	....	*Engineer*, June 17, 1927, p. 648.
14	0.91†	0.57†	Burma (Shwebo)	Kennedy	....	*Minutes of Proceedings*, Inst. C. E., p. 260, Vol. 229 (1929–30); Parker, "Control of Water."
15	0.95	0.57	Chenab, Punjab.	Lindley	....	*Minutes of Proceedings*, Inst. C. E., p. 260, Vol. 229; Punjab Eng. Congress, *Proceedings*, 1919, p. 63.
16	0.756	0.64	Sirhind, Punjab.	W. B. Harvey	....	*Proceedings* Punjab Eng. Congress, 1919, p. 58.
17	0.84	0.64	Bari Doab	Kennedy	....	Hydraulic Diagrams, Kennedy, Public Works Dept., India, 1907.
18	0.924	0.64	Penner River	J. M. Lacey	Lower	*Minutes of Proceedings*, Inst. C. E., p. 333, Vol. 229.
	0.924	0.64	Cauvery Delta	J. M. Lacey	Lower	
19	1.09	0.64	Penner River	J. M. Lacey	Upper	*Minutes of Proceedings*, Inst. C. E., p. 333, Vol. 229.
20	0.966	0.64	Cauvery Delta	J. M. Lacey	Upper	
21	0.98	0.64	Imperial Valley	Rothery	....	*Minutes of Proceedings*, Inst. C. E., p. 179, Vol. CCXVI.
22	1.26	0.64	Cauvery Delta	J. M. Lacey	Extreme. Upper	*Minutes of Proceedings*, Inst. C. E., p. 333, Vol. 229.
23	1.33	0.61	Imperial Valley	Collings	Lower	*Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 549.
24	1.83	0.61	Imperial Valley	Collings	Upper	
25	0.42	0.64	Behera Delta	R. G. Kinder	....	*Proceedings*, Punjab Eng. Congress, 1919, p. 74j.
26	‡	‡	Jamrao, Sind	W. L. C. Trench.	....	*Minutes of Proceedings*, Inst. C. E., p. 307, Vol. 223, 1926–27.

* "Limit" refers to the upper or lower limit of the data observed, which may spread over a considerable range.

† Approximate.

‡ $V_o = (1.1+)\ 0.095\ d$.

of hydraulic diagrams for the design of channels was presented by Capt. A. Garrett which deals with non-silting channels (8), and which is used extensively in the United Provinces.

In 1917, Mr. F. W. Woods proposed (2) the use of definite ratios of depth to width, based on an analysis of data from the Lower Chenab Canal System. In 1919, the results of an extensive analysis of canal dimensions of the Lower Chenab Canal, by E. S. Lindley, M. Am. Soc. C. E., was published (3). For

these canals, Mr. Lindley found a critical velocity relation such that, in Equation (1), $C = 0.95$ and $n = 0.57$. He also found a relation of bed width to depth of $B_B = 3.8\ d^{1.61}$.

In 1927, Woods (4) proposed a general formula covering velocity, average depth, mean width, and slope, as follows:

$$d_a = B_m^{0.434} \qquad (2)$$

$$V_0 = 1.434 \log_{10} B_m \qquad (3)$$

and,

$$S = \frac{1}{2 \times \log_{10} Q \times 1\,000} \qquad (4)$$

Equations (2), (3), and (4) cover not only the depth and width, but also the discharge and slope. According to them for a given discharge there is a single condition of depth, width, and slope that will produce a stable channel.

In 1928, Mr. W. T. Bottomley (9) advanced the idea that irrigation channels would be non-silting and non-scouring if the slope of the canal was of the same order as that of the parent river, regardless of the relation of width to depth and the shape of the channel. In 1930, an excellent paper on this subject (18) was presented by Mr. Gerald Lacey in which he advanced the proposition that the wetted perimeter of stable channel was a simple function of the square root of the discharge; or,

$$P = 2.668\ Q^{0.5} \qquad (5)$$

and that the shape of the section depended upon the fineness of the silt carried, coarse silt giving rise to wide, shallow sections and fine silt to narrow, deep ones. He developed the formulas,

$$Q\ f^2 = 3.8\ V_0^6 \qquad (6)$$

and,

$$V_0 = 1.17 \sqrt{f\ R} \qquad (7)$$

in which f is a silt factor, related to the diameter of the bed material by the expression:

$$f = 8 \sqrt{D} \qquad (8)$$

In Equation (8) D is in inches. From Equations (6), (7), and (8), knowing the flow, Q, in the ditch, V_0, A, and R can be computed.

Lacey also stated that the shape of a stable channel approximated an ellipse, with its major axis horizontal, the ratio of the major to the minor axis being larger as the silt became coarser. Lacey's ideas have been widely accepted in India, and extensive observations are under way to study the effect of various conditions on his silt factor, f.

The aforementioned authorities have developed their ideas almost entirely from experience in India. The result of experience on canals in Egypt is

given by Messrs. Molesworth and Yenidunia (10). They give a general formula in English units:

$$d = (9\,060\ S + 0.725)\sqrt{B_B} \quad \text{.....................(9)}$$

as developed from a careful examination of a large number of recognized good Egyptian canals. As a result of further experience, Mr. A. B. Buckley (11), develops the adjustment of Equation (9) for canals of depths of 1.6 m (5.26 ft) and less, as follows:

$$d = \frac{0.0025\ (100\,000\ S + 8)^2\ B_B}{1.62} \quad \text{...............(10)}$$

Equation (10) is in English units. In addition to the general formulas proposed by various investigators, a large number of special formulas of the Kennedy type have been developed. These formulas are listed on Table 1.

Summary of Previous Stable-Channel Formulas

The formulas developed for stable channels fall into two classifications: (*a*) Those giving an expression for velocity; and (*b*) those giving stable channel shapes. Those in the first class are similar to the Kennedy formula, Equation (1). In most of these formulas n has been taken as 0.64, the value developed by Kennedy. In all cases the value of C was constant for a given locality or canal system. Kennedy believed that C would vary with both the size and quantity of silt, but did not emphasize the effect of the quantity of silt as much as the quality, and, as a result, it has been largely lost sight of by other students of the subject. He did not believe that the value of n would change greatly.

A formula of the Kennedy type indicates that the critical velocity increases with the depth, but experience shows that as the depth is increased a velocity is finally reached at which the banks begin to erode. Kennedy believed that the limiting velocity was a matter of experience, and gave limiting values which correspond to depths of about 10 ft. This had the effect of limiting the depths of canals designed according to his rules to these values. No data on this limitation are available for the conditions under which any of the other formulas of the Kennedy type were developed.

Of the formulas of the second type, Lindley gives a relation of critical velocity to depth and bed width, but suggests no modifications for the quality or quantity of silt. Woods gives relations for mean depth, velocity, and slope, but like Lindley makes no suggestion that these relations might be influenced by the quantity or the quality of the silt. Lacey gives channel shapes and velocities. introducing the effect of the size of the silt grain, but does not consider the quantity of material to be transported.

Comparison of Critical Velocities

In order to determine what velocities could be used safely in the All-American Canal, Fig. 1 was prepared, showing the relation of depth to critical velocity, as determined from all available observations on actual ditches.

These show at a glance that for a given depth of flow there is a tremendous variation in critical velocity. The line representing Kennedy's data is shown heavier than the others. The variations range approximately from 46 to 208% of Kennedy's results, or the highest value over 450% of the lowest.

The local conditions under which most of these formulas were developed are not known in detail. In general, however, it is believed that the silt of the Nile River is finer than that of Ravi River, from which water is drawn for the Lower Bari Doab Canal, on which Kennedy's observations were made. The lower velocities found for the Egyptian canals, as compared with those given by Kennedy are, therefore, consistent with the relation of Lacey's formula (Equation (6)), that finer material results in lower critical velocities. It is also known, however, that the silt of the Colorado River and the tributary Imperial Valley canals is finer than that of the Ravi, but the critical velocities are higher in the case of the Imperial Valley canals. This is contrary to the relation given by Lacey.

Comparison of Formulas for Width-Depth Relation

A comparison similar to that of the critical velocity relations was also made of the various formulas for the relation of bed width to depth. The results are shown on Fig. 2, which gives the relation of bed width for the principal formulas and some of the data. The Woods' formula was expressed in terms of mean width, and has been changed to terms of bed width by assuming side

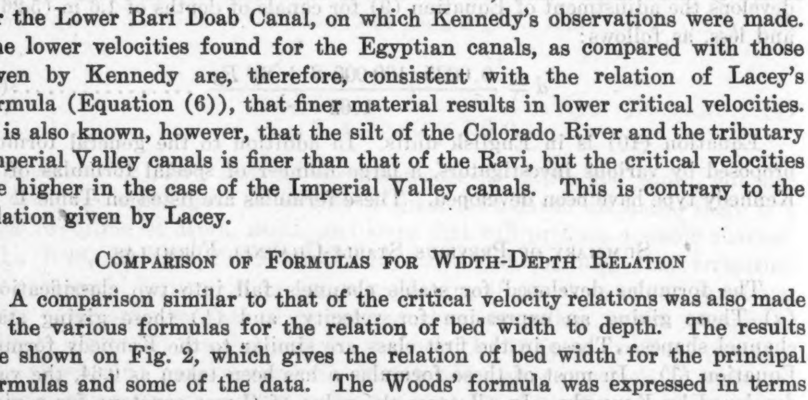

FIG. 2.—BED WIDTH-DEPTH RELATION FOR A NON-SILTING NON-SCOURING CANAL.

slopes of 1 on 2. The data for channels as proposed by Lacey (using side slopes of 2 to 1) for three sizes of material, are also shown. The finest of these, 0.0025 in. in diameter, is for material roughly corresponding in size to that composing the bottom of the Imperial Valley canals.

The Punjab (Kennedy) data are computed from those given by Kennedy for the Lower Bari Doab Canal, using vertical side slopes, as reported by him.

Data are also given on canals in the Godavari Western Delta and some values obtained from canals in the Imperial Valley. The data from Egypt were in the form of a general equation by Molesworth and Yenidunia, which is independent of the slope, and an equation which is dependent on the slopes, four slopes being given. For depths less than 1.62 m (5.32 ft), a modification of the Molesworth-Yenidunia, formula given by Buckley has been used, based on data which have been collected since the other formula was proposed. The Depuit Section, said to be widely used in Egypt, is also shown.

These data for the bed width-depth relation show even greater variation than the depth critical-velocity relations shown on Fig. 2. For a 5-ft depth, the Molesworth-Yenidunia formula without the slope factor, gives a bed width of 6.4 ft and the Lindley equation gives 50.0 ft, or a ratio of maximum to minimum of 781 per cent. Some of the Imperial Valley data indicate even higher ratios than those given by the Lindley formula. The wide range does not seem to be due to variation in the size of the silt because, although the Egyptian data are believed to be for finer silt than the Indian data of Woods and Lindley, most of the Imperial Valley data, which are also for fine silt, give even higher bed width-depth ratios than those of either Lindley or Wood.

Factors Affecting Stable Channel Shapes

As a result of the wide range of critical velocities and bed width-depth relations found on the canals in the different parts of the world, and the lack of any readily apparent consistency in the variations, it was clear that if the factors controlling this variation could not be determined it would not be safe to adopt any of the relations given by existing formulas for the design of the sections of the All-American Canal. Although these formulas no doubt provide workable relations for the conditions for which they were developed, these conditions have not been delineated sufficiently to enable them to be applied elsewhere. In general, also, they were developed empirically from a very limited range of conditions, and in most cases they omit important factors from consideration.

To develop a rational design for the sections of the All-American Canal, it was necessary, therefore, to attempt to go back to the fundamentals and try to make an analysis of the factors controlling the shape of a stream channel in erodible material, and their relations to each other.

The following is a list of factors that may enter into a determination of stable channel shapes: (*a*) Hydraulic factors (slope, roughness, hydraulic radius or depth, mean velocity, velocity distribution, and temperature); (*b*) channel shape (width, depth, and side slopes); (*c*) nature of material transported (size, shape, specific gravity, dispersion, quantity, and bank and subgrade material); and, (*d*) miscellaneous (alignment, uniformity of flow, and aging).

In arriving at a rational solution of the problem of stable channels it is necessary to consider all these factors, and to determine as accurately as possible which of them are of major importance, and which are minor or negligible. By determining first the relation between major factors, it may be

possible later to study the effect of minor factors, but until the major relations are known, the data available are only a collection of miscellaneous facts of limited value.

Of the hydraulic factors, the slope, roughness, hydraulic radius, and mean velocity are interdependent and with reasonable certainty their relation is known quantitatively through the ordinary velocity formulas. It is true that the effect of the movement of material in suspension and by traction upon the roughness is not definitely known and more information on this point is needed, but compared to the uncertainty in other phases of the problem, the relations of these four items is so well known that for the purposes of this study further investigation along these lines was probably unjustified. The relation between these factors and scouring or transportation of solids in channels is not well established and must be studied further.

As will be shown subsequently, it is believed that the velocity distribution, as well as the mean velocity, is of primary importance to the problem and that it, together with the channel shape factors of width and depth, exercise an important influence on stable shapes. The side slopes are relatively unimportant except as regards sloughing.

Temperature has been suggested as having an important effect because it influences the viscosity of the water and, consequently, the rate at which solid particles settle. That temperature might have some effect cannot be questioned, but it is probably small, from the standpoint of stable channel shapes. Most of the data were collected in warm countries, comparable to the locality of the All-American Canal, and the temperature variation although it might cause some difference in settlement rate would not, ordinarily, be enough, when averaged over the year, to cause major effects. For many sizes of silt the effect of temperature on settling rate is small. Moreover, it is possible that the tractive force, which is not appreciably affected by temperature, is the most important factor in stable channel shapes, and, therefore, temperature effects are relatively unimportant. In any event temperature data, which would enable an analysis of its effect to be made, are not available.

Nearly all students of the problem have admitted that the size of the material transported is of major importance. The shape no doubt has an effect, but it is believed to be of secondary importance as laboratory experiments show that angular particles are moved by only slightly higher velocities than rounded ones. In any event, no data on it are available for any of the localities, so that its influence could not be investigated, even if it was desirable to do so. Specific gravity of the transported material also has its effect, but since it rarely varies much from 2.65 it is of secondary importance. No data on this subject would be available, even if it were desirable to study them. The dispersion of the material by virtue of the electrical charges carried by the particles is important in some phases of sedimentation, but it is probably active only in the case of very fine material which is ordinarily not much of a factor in stable channel sections. In this case, again, no data for further study are available.

The quantity of solids in motion is an important factor in the shape of stable channel shapes, and has not received the attention that its importance

warrants. Cases illustrating its importance are numerous. For example, it is a common occurrence in some irrigation systems to have the upper section of the ditch fill during periods of high silt content in the streams from which they draw, and for this fill to scour out later during periods of clear-water flow. In other words, the ditch is unstable, at times being unable to transport all the material brought to it, hence filling up, and at other times transporting more material than brought to it, and, therefore, cutting down its bed. Over long periods the ditches are approximately stable because the two actions counteract one another. Another common example is the change from unstable to stable condition which results when an effective sand trap is applied to a ditch that is becoming filled. There are also numerous cases in which the channel of a natural stream is stable but begins to scour severely when a dam is built on it and cuts off the supply of solid material which formerly came down to renew that which was moved forward by the flowing water.

One of the most important factors controlling stable channel shapes is the nature of the material composing the banks and subgrade. If these materials are resistant to scour, higher velocities can be used than if the material is friable. Alignment is another factor to be considered, because bank scour is more likely to occur on curves. If a canal is apt to be operated a large proportion of the time at part capacity, this must also be considered in the design.

Another factor that influences the stability of an irrigation channel is what is commonly termed "aging." After water has run for some time in a channel, the particles composing the bed arrange themselves in such a manner that they are more difficult to move than when the water is first turned in. If the water is silty, this material forms a kind of weak cement which binds the bed material together and makes it more resistant.

In addition to the items listed herein under the heading "Factors Affecting Stable Channel Shapes," another set of relations enters into the selection of the best channel section in any instance, which depends upon the conditions which the canal is designed to meet.

Canals for conveying water for irrigation or power are usually designed to meet one of three sets of conditions. The first type is encountered when it is desired to use the lowest practicable velocity, in order that the slope may be reduced to a minimum. In the case of power canals this is done to obtain the greatest feasible power head, and, in irrigation canals, it is done to enable the ditch to command as much irrigable area as possible for a given length. A second type of conditions is met in both power and irrigation canals where it is desired to reduce the size of the canal to a minimum, in order to make the cost as small as possible without making the slope steeper than necessary. This requires that the velocity be made as great as can be carried without scouring the banks or bed. A third condition is met in irrigation canals where it is desired to carry the ditch on an alignment that has a slope as steep as possible, in order to reduce the cost of drops. The first of these conditions aims at securing the minimum practical velocity within the limitations of cost and silting. The second aims to secure the highest velocity that the ditch will stand with a shape which will convey the water with a reasonable loss of head. The third aims to dissipate as much head as

possible by making the canal wide and shallow, thus reducing the hydraulic radius to a minimum and the slope for a given velocity to a maximum. The group into which any particular canal falls, therefore, indicates limitations which are likely to control the best channel shape, which must not be exceeded while still being subject to the influence of the aforementioned factors.

Conditions Required for Stable Channels

For a channel to be perfectly stable, it must not fill or scour on either the banks or bed. The banks must also be stable against sloughing or sliding. To meet the non-filling requirement, the velocity must be enough to flush away all the solid material brought into the section by the flowing water. To fulfill the non-scouring requirement, the velocity at the bed or at the banks must not be great enough to scour the material of which they are composed. To determine a stable section for a set of conditions it is necessary to determine the various relations which will cause velocities at the banks and along the bed that will bring about these conditions.

The silt carried into a section of canal may be composed entirely of fine material, which is easily moved by the water, or it may be composed entirely of coarse material, which is moved only at relatively high velocities. Usually, however, it has a graded composition varying from coarse to fine. If all the material is very fine, ordinarily it offers little practical difficulty because the velocities required in the ditch to meet conditions of economy are sufficient to keep it in motion. If the material is graded, the fine material moves in suspension and the coarse material is rolled along the bed. If all the material is coarse, all of it may be dragged along the bed, and little if any be carried in suspension. Since the quantity of the bed material that can be moved depends upon the velocity near the bed, to obtain a stable channel, the velocity along the bed must be greater for larger bed loads. This may require a higher velocity along the bed than the material in which the channel was built would stand from clear water, the entire energy of the water on the bottom being expended in dragging along the bottom the material which has been brought down by the water from above. However, if the velocity along the bottom exceeds that necessary to move the bed load, it will act on the subgrade of the channel. To have a stable channel, the subgrade material must be sufficiently tenacious to resist this scour. Summarizing this relation, it may be stated that the velocity along the bottom of a stable channel must be sufficient to move the quantity of material supplied to it, but not so great as to scour the subgrade.

The material composing the banks of the canal is acted upon by two forces tending to produce motion. One of these is gravity, which tends to make the material roll or slide down the sides of the ditch. The effective gravity force is the component that acts downward along the side siopes of the ditch. The other force acting is that due to the motion of the water through the canal, which tends to drag or to push the material in a down-stream direction. The magnitude of this force depends upon the velocity adjacent to the bank. The force of gravity and the force of the stream both act together when water

is flowing in the canal, and when the resultant of the two forces is sufficient to dislodge material from the sides, it moves in a diagonal direction to the bed of the stream, or if fine enough, is carried off in suspension by the water. The slope of the bank must be sufficiently flat so that the component along it, of the force of gravity, when combined with the force of the water, is insufficient to dislodge the particles. Since flat side slopes cause a smaller component of gravity, therefore, they have less tendency to scour from this cause.

Stable Channels for Clear Water

The simplest cases involving the determination of stable channel sections are those required to convey clear water. When the water carries silt in suspension or drags a load along the bottom, there are added complications. Therefore, the simplest cases, with clear water, will be considered first. When the smallest practical slope is desired, it is usually considered that the cheapest channel is secured when the wetted perimeter is least in proportion to the area. In trapezoidal channels this occurs with a ratio of bed width to depth ranging from 2.0 to 0.472 for side slopes between the vertical and 1 on 2. These values give the most efficient hydraulic section, but since this consideration neglects any excavation above the water line the flattest slope for a given quantity of excavation for a channl in cut is given by cross-sections where the ratio of bed width to depth is less than the foregoing. Thus, maximum economy will result from very low $\frac{B}{d}$ -ratios. In earth, however, experience has shown that such channels are not stable.

A stable channel for clear water must have banks with sufficiently flat side slopes to keep the material from sloughing or rolling in, and sufficiently low velocities to keep the banks and bed from scouring. As previously stated, since the material on the banks is acted upon by the force of gravity, as well as that due to the motion of the water, it will not resist as high a force from the motion of the water as the bottom, where gravity does not tend to produce motion.

If the mean velocity in a narrow, deep channel is low enough so that the forces acting on the side material are insufficient to move it, and the sides are stable from sloughing, the channel will be stable. In other words, narrow, deep channels can be used with clear water and low velocities. Ordinarily, however, considerations of cost prevent the use of the large canals necessary to produce low velocities. For such a channel a cross-section must be selected that will give velocities along the bottom which will not move the bottom material, and velocities along the sides which will not move the side material. Since the side material, due to the action of gravity, will move at a lower velocity than that on the bottom, to obtain the maximum possible mean velocity without scour the velocity along the sides must be enough less than that along the bottom to offset the gravity effect. This reduction of side velocity, as compared with bottom velocity, is secured by increasing the ratio of bed width to depth.

In Fig. 3 are shown the velocity distributions in a number of rectangular channels having the same cross-sectional area. Most of these data were secured from the results of the experiments of D'Arcy and Bazin. The velocities are indicated by the "isovels" (also called "isotacks"), or lines of equal velocity expressed in terms of the mean velocity. (These data were obtained with different discharges for many of the examples. Although it is probable that the positions of the isovels would change somewhat with different velocities, such changes would be relatively small and would not change the main relations.) Since the areas of all water cross-sections are equal, the lines giving the same ratio to mean velocity in all the diagrams represent the same

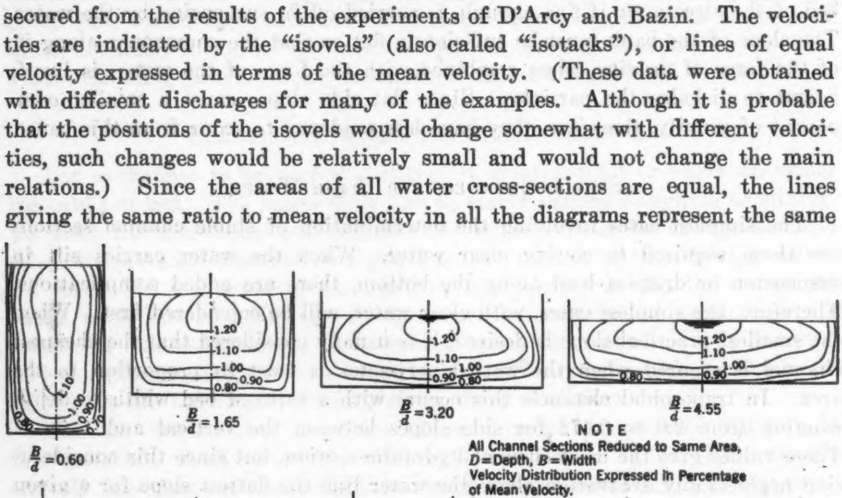

FIG. 3.—RELATION OF WIDTH TO VELOCITY DISTRIBUTION IN RECTANGULAR CHANNELS.

velocity for any given discharge. A study of the velocity distribution in these sections will show that high velocities extend closer toward the sides in the narrow, deep cross-sections than in the broad, shallow ones. The science of hydrodynamics has not yet progressed to the point where the relation between the velocity distribution adjacent to a surface can be related quantitatively to the drag of the water along the surface or to the velocity "gradient" adjacent to the surface; but progress along this line is rapid, and the near future may bring sufficient advancement in this field to enable more exact analysis to be made. For the present, however, it is sufficient to state that when the high velocities extend close to a surface the pushing or dragging force of water on the surface is greater than if these velocities close to the surface are low.

In a very narrow, deep section, the velocities close to the sides are as high as, or higher than, those close to the bottom. If the velocity in such a channel is increased gradually, due to the added force of gravity on the side material, motion would occur first on the sides.

The design of a channel to convey clear water where it is desired to obtain the smallest practicable slope, therefore, consists, from the hydraulic

standpoint, of securing the smallest ratio of bed width to depth that will not produce scour on the sides, provided such ratio is not less than that which will give the smallest wetted perimeter for a given earthwork quantity. This latter qualification will probably rarely control. The design of a channel for the second type of conditions, where the highest practicable mean velocity is to be secured, is obtained by proportioning the ratio of width to depth so that the forces tending to produce movement, both on the sides and bottom, are the maximum they will stand without motion. For canals in the third class, where it is desired to make the ditch as steep as possible without producing scour, it is customary to make the section wide and shallow in order to reduce the hydraulic radius and thus lower the velocity. For such very wide ditches the scour would be greatest on the bottom, and this condition would control the slope that might be used. Theoretically, there is no limit to the slope a ditch might be given because the velocity could be reduced to any desired value by making it sufficiently wide and shallow. As a practical matter, however, it has been found that when the depth is made very small, the irregularities of construction are such that scour starts in the slightly deeper parts of the channel and enlarges them, causing a progressively greater concentration and scour until the beneficial effect of the widening is lost. This action has been noted in ditches with steep slopes 8 to 10 ft wide and 0.6 to 0.8 ft deep.

Channels Carrying Solids in Suspension

It is quite generally agreed that material can be carried in suspension by a stream because of the vertical currents that occur in flowing water and carry the solid particles upward at a greater rate than the force of gravity causes them to fall. A promising hypothesis for the capacity of a stream to transport material in suspension, therefore, should be that the capacity is proportional to its turbulence, which, in turn, is probably proportional to the energy expended. The concentration of a given quality of solids which a stream could support, therefore, would be proportional to the energy expended per unit of volume of the water. This energy is proportional to the rate of fall of the water, which is equal to the product of the velocity and the slope.

This is a kind of over-all relation, however, and silting may occur in one part of a ditch cross-section while other parts may be scouring. Because the velocity near the edge of a stream flowing in a trapezoidal section is low, in a channel carrying silt in suspension there is a tendency to deposit at that point. This is aided by the growth of vegetation, and usually a berm is formed which creates steeper sides to the section than were originally constructed. This material is quite resistant to scour, and forms more or less evenly even in ditches where there are high velocities. This action is not ordinarily very detrimental, and is often anticipated and allowed for by computing the capacity of the channel with the slopes which it is expected the silt will cause, rather than the slopes to which it is first excavated.

It is probably not feasible to prevent entirely the deposition of suspended material along the edges of channels in earth, although it may be reduced by using higher velocities. The greatest difficulties from suspended matter

occur in ditches where the slopes are so slight that the energy being dissipated in the water is insufficient to prevent deposit. The remedy in these cases is to increase the velocity, or to remove the suspended material in some kind of desilting device.

Channels Carrying Bed Load

Irrigation channels frequently carry considerable solid matter by dragging or pushing it along the bed, with either clear or silt-laden water flowing above. The quantity of material depends upon the velocity near the bottom of the channel. Higher velocity is probably necessary to move the same quantity of coarse material as fine material. If a channel is supplied with a heavy bed load, in order to be stable it must move this load along; otherwise, the channel will become filled. This requires a high velocity along the bottom, as compared with a channel carrying clear water. For a given quality of material in the banks, the velocities that could act on the banks in the two cases would be the same. To be stable, the channel carrying bed loads, therefore, should have a higher velocity along the bed, but the same velocity along the banks, and this could only occur with a wider, shallower section. Heavily loaded channels in easily scoured material therefore, should have high ratios of bed width to depth. If the banks of the loaded channel are of material which is resistant to scour, the ratio of bed width to depth can be less than in friable material without scouring the banks.

Colloids

Colloids carried in the water exercise a considerable effect on the shape of the channel cross-section. They cement the fine particles which collect along the sides of the ditch and are responsible for the vertical or nearly vertical banks which exist in many canals. This colloid cemented material along the sides is more resistant to scour than the size of material would indicate, and thus permits higher velocities along the banks than would otherwise be allowable. To a certain extent also colloids may cement the particles composing the bed and make it more resistant to scour. It is believed, however, that some of the effects ascribed to colloids are really due to the presence of a high silt load. The ability of the canals of the Imperial Valley, which are constructed in fine silt, to carry velocities of 4 or 5 ft per sec without scour has been ascribed to colloids, but the writer believes that a large part of it is due to the presence of the high silt load. When these canals are supplied by the All-American Canal with desilted water, their beds will scour considerably, and this would not be prevented by the colloids.

Recent Conceptions of Flow Not Used

To engineers who are familiar with the latest theories of stream mechanics and hydrodynamics, the foregoing analysis of the factors controlling stable channel shapes may seem somewhat crude, and in ignoring the drag theory of bed-load movement and the conception of velocity gradient, it may appear that the writer has not taken advantage of the best available information.

In making the study outlined herein, the most recent pertinent literature in stream mechanics and hydrodynamics has been analyzed to determine all the material that was applicable to the problem. A list of these references is included in the "Bibliography" contained in this paper. Since many engineers are not familiar with recent ideas, however and since a knowledge of them is not necessary to understand the relations developed regarding stable channel shapes, it was believed to be better to explain these relations in terms of conceptions with which all engineers are familiar, rather than to make it unnecessarily confusing to some by adding the other new ideas.

Agreement of Suggested Relations with Observed Data

The relations suggested in this paper seem to agree with the observed data, as shown on Figs. 1 and 2. The critical velocity shown for the Nile River on Fig 1 is much less than that found by Kennedy in India and also less than that for the Imperial Valley canals. The quantity as well as quality of silt is an important factor in these cases. The silt in both the Nile and Colorado Rivers is fine, but the Imperial canals require a much higher velocity than the canals of Egypt because the quantity of silt is much greater in the Imperial canals. The critical velocity for the canals observed by Kennedy is higher than those in Egypt, probably because the particles moved on the bed were larger, and, therefore, required higher velocities to move them. The Imperial canals require more velocity than the canals observed by Kennedy, although the latter have coarser loads, because the velocity required to transport the immense bed load of the fine Imperial Valley sand is greater than that necessary for the lesser quantity of coarser sand of the canals mentioned by Kennedy.

A similar agreement of the relations previously discussed in found in the data on bed width-depth ratios as shown on Fig. 2. In the canals of Egypt the velocities are low and, therefore, the velocities along the sides, although relatively high because the channels are narrow and deep, are still below those that will move the side material. The bed load is fine and small in quantity and, therefore, the low velocities along the bottom are sufficient to move it all.

The Indian canals shown on the diagram (Fig. 2) carry medium loads of rather coarse material and, therefore, require rather wide sections. The Imperial Valley canals carry immense loads of fine sand, which require high velocities to transport. In order that the velocities along the sides may be low enough so that the banks do not scour, the bed width-depth ratio must be high. In three of the four canals on which data are available, this relation is higher than that indicated by the equations of either Woods or Lindley. These had readily erodible sides. The fourth, which had a lower bottom width-depth ratio, had sides composed of material which had considerable resistance. The three sections with easily erodible banks gave widths considerably greater than is indicated by Lacey's formula for the type of silt which they contained. It is believed that Lacey's formula was based on data from canals which carry loads of considerably less magnitude.

Stable Channel Shapes

The only investigator who has attempted a closer definition of the shape of stable channels than the bed width-depth relation is Lacey. He states (18):

"That natural silt-transporting channels have a tendency to assume a semi-elliptical section is confirmed by an inspection of a large number of channels in final regime and an examination of cross-sections of discharge sites of rivers in well-defined straight reaches of known stability."

He concluded that stable channels would be semi-elliptical, with the major axis horizontal, and with the ratio of the major axis to the semi-minor axis depending on the nature of the silt carried, being greater for coarser silt.

The results of the writer's investigations do not support this conclusion. A convenient way of comparing channel cross-sections is by means of a ratio, which may be called the "form factor," between the area of the channel section, up to the water surface, and the area of the enclosing rectangle. For an ellipse this ratio would be $\frac{\pi}{4} = 0.79$; for a parabola, 0.67; for a triangle, 0.50; and for a rectangle, 1.00. A study of a large number of cross-sections channels carrying a heavy load of graded silt, ranging from colloids to fine observed by Kennedy were reported to have practically vertical sides and horizontal bottoms, which would give a form factor of 1.00.

Just what shapes for stable channels are produced by all varieties of conditions has not yet been determined, but the writer has observed that for channels carrying a heavy load of graded silt, ranging from colloids to fine sand, the sections have nearly horizontal beds composed of the fine sand and nearly vertical sides of silts and clays. Such channels have form factors of about 0.90. This is the condition on the canals of the Imperial Valley. This difference between the composition of the bed and bank material has also been observed in India. Similar conditions result in channels carrying considerable bed load and a moderate quantity of fine silt. For a channel carrying water containing a small quantity of silt at high velocity in a material containing a considerable number of cobbles, the cross-section is distinctly saucer-shaped, most of the section being covered with cobbles, but with a small silt berm at each edge. This condition was observed on some of the ditches on the Uncompahgre Project, in Colorado, and in the San Luis Valley, in Colorado. One ditch that was carefully measured, had a form factor of 0.85. It is believed that further study will disclose typical shapes for a number of common conditions and the reasons therefor.

Variation in Discharge

The design of the best section for many canals is complicated by variation in the flow they are to carry. Some canals fill at one time and scour out at another, a substantially stable channel resulting from the balance of scour and fill. Many of the data which can be secured on stable shapes are complicated by this discharge variation. No rules can be given for the treatment of such cases. Until the problem of the simple case of relatively uniform flow is obtained, the more complex case of variable flow can be attacked only

by the application of engineering judgment based on a knowledge of existing conditions and available information regarding shapes required for uniform flow.

ACKNOWLEDGMENTS

In the investigation leading to this paper many discussions of the problem were studied and many helpful suggestions were thus secured. With these the writer has combined his own ideas. It is not always possible to state definitely which were original and which were secured from literature. The papers on the subject which have been found most useful have been incorporated in the "Bibliography." To the authors and discussers of those papers the writer is particularly indebted. He is also indebted to C. A. Wright, Assoc. M. Am. Soc. C. E. and J. C. Stevens and S. P. Wing, Members, Am. Soc. C. E., for many helpful suggestions.

As previously stated, this paper gives the results of studies for the selection of stable channels for the All-American Canal. This canal is being designed by, and constructed under, the direction of the U. S. Bureau of Reclamation. All designs and investigations of the Bureau of Reclamation are under the direction of J. L. Savage, M. Am. Soc. C. E. All engineering and construction work is under the general direction of R. F. Walter, M. Am. Soc. C. E., and all activities of the Bureau are under Elwood Mead, M. Am. Soc. C. E. The writer wishes to express his appreciation to the authorities of the Bureau of Reclamation for permission to publish these data.

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AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

TRUSS DEFLECTIONS: THE PANEL DEFLECTION METHOD

By Louis H. Shoemaker[1], M. Am. Soc. C. E.

Synopsis

The purpose of this paper is to describe a new method for computing the deflections of trusses. In principle, the method is similar to that of computing the deflections of beams, use being made of the distortions of the individual panels. In this manner, the deflections of all the panel points of a truss can be computed with a considerable saving in work over other analytical methods. Therefore, it offers material advantages in computing the reactions of statically indeterminate structures.

Application of the Method

To compute the deflection of a point of a beam of constant moment of inertia with reference to a tangent to the elastic curve of that beam, the sum is taken of the products of the angular change of every vertical section, multiplied by its distance from the point. If the moment of inertia of the beam is not constant, it is necessary to divide the beam into longitudinal sections of lengths corresponding to the different moments of inertia. In this case, the deflection of the point is computed by taking the sum of the vertical distortions of the different longtitudinal sections, plus the sum of the products of the angular distortion of each section, multiplied by the distance of the section from the point. It is evident that the same method can be used to compute the deflections of a truss by utilizing the distortions of the individual panels of the truss.

To apply the method to a truss, a convenient point is selected as the origin, from which all deflections are measured. The vertical and horizontal deflections and angular distortion of every panel are computed. In Fig. 1, *ABCD* represents any panel of a truss. The origin of deflections is considered as being at some panel point to the right. The point, *A*, and the side, *AB*, are taken as reference point and direction from which panel distortions are measured. The triangle, *ABC*, will be called the deflection triangle,

Note.—Discussion on this paper will be closed in February, 1936, *Proceedings*.

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and C, the deflection point. The point, D, will be called the secondary point. The vertical and horizontal deflections of C are caused by the changes of length of the members of the triangle, ABC. The vertical deflection at D is the vertical deflection at C plus or minus the change of length of CD, which is also a member of the deflection triangle of the panel to the left. The angular distortion of the panel is the angular rotation of CD, due to

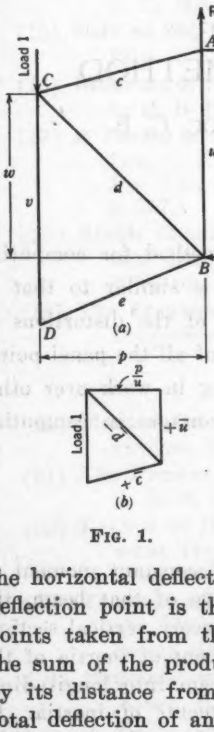

FIG. 1.

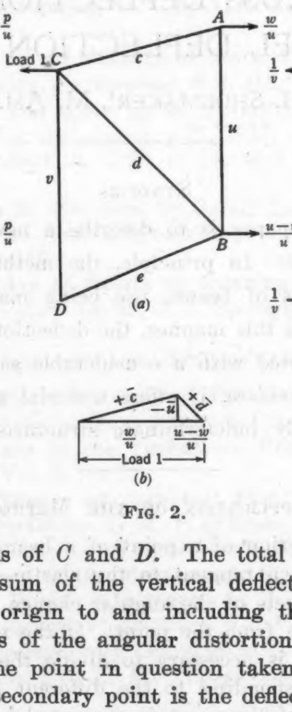

FIG. 2.

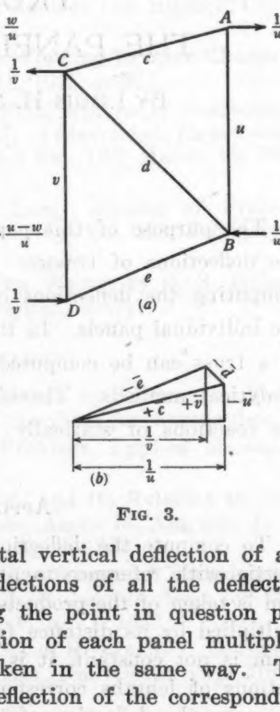

FIG. 3.

the horizontal deflections of C and D. The total vertical deflection of any deflection point is the sum of the vertical deflections of all the deflection points taken from the origin to and including the point in question plus the sum of the products of the angular distortion of each panel multiplied by its distance from the point in question taken in the same way. The total deflection of any secondary point is the deflection of the corresponding deflection point plus or minus the change of length of the connecting vertical.

NOTATION

The following notation is introduced in this paper:

c = length of a chord member of a deflection triangle; c' = change in length, $+ c'$, denoting an increase and $- c'$, denoting a decrease; $\bar{c}$ = the stress in Member c resulting from a load of 1, or a bending moment of 1, applied to the panel;

d = length of a diagonal member of a deflection triangle; d' = a change in length, $+ d'$ denoting an increase and $- d'$ denoting a decrease; $\bar{d}$ = the stress in Member d resulting from a load of 1, or a bending moment of 1, applied to the panel;

e = length of a chord member at a secondary point; e' = a change in length, $+ e'$ denoting increase and $- e'$ denoting decrease;

n = number of panels, counted from the origin;

p = length of a panel;
u = length of a vertical member of a deflection triangle; u' = change in length, $+ u'$ denoting an increase and $- u'$ denoting a decrease; $\overline{u}$ = the stress in Member u resulting from a load of 1, or a bending moment of 1, applied at the panel;
v = length of a vertical member between a deflection point and a secondary point; v' = change in length, $- v'$ denoting a decrease, and $+ v'$ denoting an increase, in length;
w = length of a vertical projection of a diagonal member;
x = number of panels from the origin to any deflection point or secondary point between the origin and the nth panel point;
y = vertical distance of any panel point n panels from the origin above or below a deflection point x panels from the origin, $+ y$ denoting distance downward and $- y$ distance upward from the xth panel point.
A = cross-section area;
E = modulus of elasticity;
H_1 = horizontal deflection of Deflection Point C with respect to Point A; H_T = total horizontal deflection of a deflection point or a secondary point, distant n panels from the origin;
L = length of a member:
R = angular distortion of a panel with reference to Member AB;
S = total axial stress applied to a member;
T = a subscript denoting "total";
Δ_1 = vertical deflection of Deflection Point C with reference to Point A; $\Delta_2 = \Delta_1 - (\pm u')$ = vertical deflection of Deflection Point C with reference to Point B; $\Delta_3 = \Delta_1 + (\pm v')$ = vertical deflection of Secondary Point D with reference to Point A; $\Delta_4 = \Delta + (\pm v') - (\pm u')$ = vertical deflection of Secondary Point D with reference to Point B;
δ = change in length, c', d', etc.

General Formulas for Panel Distortions

In developing formulas for panel distortions, the method of internal work will be utilized. To derive the formula for the vertical deflection at Point C, refer to Fig. 1, thus: by proportion, $\pm \overline{c} : 1 :: c : u$; $- \overline{d} : 1 :: d : u$; and $+ \overline{u} : 1 :: w : u$. Therefore, $\overline{c} = \frac{c}{u}$; $\overline{d} = - \frac{d}{u}$; and, $\overline{u} = \frac{w}{u}$, respectively, and:

$$V_1 = \frac{c}{u}(\pm c') - \frac{d}{u}(\pm d') + \frac{w}{u}(\pm u') = \frac{\pm (c'c) - (\pm d'd) + (\pm w u')}{u} \ldots (1)$$

The horizontal deflection at C is derived by reference to Fig. 2, thus: $\overline{c} : \frac{w}{n} :: c : p$; $\overline{d} : \frac{u - w}{u} :: d : p$; and, $- \overline{u} : \frac{w}{u} :: u - w : p$. Therefore, $\overline{c} = \frac{c\,w}{p\,u}$; $\overline{d} = \frac{d\,(u - w)}{p\,u}$; and, $\overline{u} = - \frac{w\,(u - w)}{p\,u}$, respectively, and,

$$H_1 = \frac{[(\pm c'\,c) - (\pm d'\,d) + (\pm w\,u')]\,w}{p\,u}$$

$$+ \frac{(\pm d'\,d)}{p} - \frac{(\pm w\,u')}{p} = \frac{w\,\Delta}{p} + \frac{(\pm d'\,d) - (\pm w\,u')}{p} \ldots\ldots (2)$$

The angular distortion of Member CD is derived by reference to Fig. 3, thus: $\bar{c} : \frac{1}{u} :: c : p$; $-\bar{e} : \frac{1}{v} :: e : p$; $-\bar{u} : \frac{1}{u} :: u - w : p$; $\bar{v} : \frac{1}{v} :: v - w : p$; and, $-\bar{d} : \frac{1}{u} - \frac{1}{v} :: d : p$. Therefore, $\bar{c} = \frac{c}{p\,u}$; $\bar{e} = -\frac{e}{p\,v}$; $\bar{u} = -\frac{u-w}{p\,u}$; $\bar{v} = \frac{v-w}{p\,v}$; and, $\bar{d} = -\frac{d\,(v-u)}{p\,u\,v}$, respectively, and:

$$R = \frac{c}{p\,u}(\pm c') - \frac{e}{p\,v}(\pm e') - \frac{u-w}{p\,u}(\pm u') + \frac{v-w}{p\,v}(\pm v') - \frac{d\,(v-u)}{p\,u\,v}(\pm d')$$

$$= \frac{1}{p}\left[\frac{(\pm c'c) - (\pm d'd) + (\pm w u')}{u} + \frac{(\pm d'd) + (v-w)(\pm v') - (\pm e'e)}{v} - (\pm u')\right]$$

$$= \frac{1}{p}\left[\Delta_1 + \frac{(\pm d'd) - (\pm e'e) - (\pm w v')}{v} + (\pm v') - (\pm u')\right] \ldots\ldots (3)$$

The total deflections are expressed, as follows:

$$\Delta_T = \sum_{x=0}^{x=n}\Big[\Delta + R\,p\,(n-x)\Big] \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

and,

$$H_T = \sum_{x=0}^{x=n}\Big[H - R\,(\pm y)\Big] \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

It is evident from the manner in which Equations (1) to (5) have been derived that the resultant signs of Δ, H, and R, have the following significance: $+\,\Delta$ indicates deflection inward and $-\,\Delta$, deflection outward with respect to the truss; $+\,H$ indicates deflection away from the reference point and $-\,H$, toward the reference point; and, $+\,R$ indicates left-handed rotation if acting to the left of the reference side and right-handed rotation if acting to the right of the reference side, and $-\,R$ is the reverse.

The formulas for panel distortions are very simple in form and composition. There are five terms involved for each panel, the products, $c'c$, $d'd$, etc. When these terms have been computed, the evaluation of the formulas to determine the total deflections, is a simple matter. Two short examples are offered to illustrate the application of the method. The loads indicated are in kips, 1 kip being equal to 1 000 lb.

Example 1.—Deflections of a Simple Truss.—In Table 1 the deflections for the 180-ft truss shown in Fig. 4 are calculated. The deflection points are 0, 1, and 3. The values of Δ are the deflections upward of 0 from 2, 1 from 3, and 3 from 5. The deflection of Point 2 is obtained by adding algebraically the change of length of Member 1-2 to the deflection of Point 1. The signs of Δ for 1 and 3 are minus as the deflections are outward, and

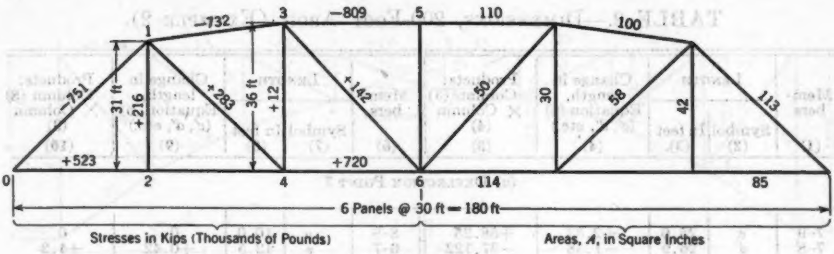

FIG. 4.

that of 0 is plus for an inward deflection. In computing the changes of length, the formula,

$$\delta = \frac{S\,L}{1\,000\,A} \qquad (6)$$

has been used, this being the variable factor. The constant factor, $\frac{12}{30\,000}$, is applied in the final calculations. The calculated deflections are upward from Panel Point 5. These are transposed in the last two columns to give deflections downward from Point 0, in inches.

TABLE 1.—DEFLECTIONS OF A 180-FOOT SIMPLE TRUSS.

Deflection points	Members	Length: Symbol	Length: In feet	Change in length, Equation (6) (c', d', etc.)	Products: Column (4) × Column (5)	Vertical deflection, Δ, in inches × $\frac{30\,000}{12}$	Values of n	$Rp\,(n-x)$: Point 3 (R = − 11.4 and x = 1)	$Rp\,(n-x)$: Point 1 (R = − 10.9 and x = 2)	Summation: Vertical deflection, Δ, in inches × $\frac{30\,000}{12}$	Summation: $Rx\,(n-x)\Sigma p$	Total Deflection, V_T: In inches × $\frac{30\,000}{12}$	Total Deflection, V_T: Transposed	Total Deflection, V_T: Multiplied by $\frac{12}{30\,000}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
5.6						0						0	2 366	0.95
3	3–5	c	30	−220.8	−6 624	−354	1	0	0	354		354	2 012	0.81
	3–6	d	46.8	+131	+6 127									
	4–6	s	30	+189.3	+5 679									
	5–6	u	36	0	0									
	3–4	v	36	+14.4	+518									
4	...	w	36			−340				340		340	2 026	0.82
1	1-3	c	30.4	−222.5	−6 764	−426	2	−342	0	780	342	1 122	1 244	0.50
	1–4	d	43	+210	+9 030									
	2–4	s	30	+184.5	+5 535									
	3–4	u	36	+14.4	+446									
	1–2	v	31	+159.3	+4 938									
2	...	w	31			−267		−342		621	342	963	1 403	0.57
0	0–2	c	30	+184.5	+5 535	+734	3	+684	+327	1 355	1 011	2 366	0	0
	0–1	d	43	−286	−12 300									
	1–2	u	31	+159.3	+4 938									
	...	w	31											

Example 2.—Horizontal Reactions of a Two-Hinged Arch.—In Tables 2 and 3 the horizontal reactions of the 200-ft arch shown in Fig. 5 are calculated. The stresses in the half arch have been computed for a load of unity applied horizontally at 0. The formula,

$$\delta = \frac{S\,L}{A} \qquad (7)$$

TABLE 2.—DIMENSIONS, 200-FOOT ARCH (EXAMPLE 2).

Members (1)	Length: Symbol (2)	Length: In feet (3)	Change in length, Equation (6) (c', d', etc.) (4)	Products: Column (3) × Column (4) (5)	Members (6)	Length: Symbol (7)	Length: In feet (8)	Change in length, Equation (6) (c', d', etc.) (9)	Products: Column (8) × Column (9) (10)
(a) Deflection Point 7									
7-9	*c*	25.0	+2.33	+58.25	8-9	*u*	10.0	0	0
7-8	*d*	26.9	−1.38	−37.122	6-7	*v*	12.5	+0.42	+4.2
6-8	*e*	25.0	−2.51	−62.75	...	*w*	10.0	......	......
(b) Deflection Point 5									
5-7	*c*	25.0	+2.14	+53.5	6-7	*u*	12.5	+0.42	+5.25
5-6	*d*	28.0	−2.61	−73.08	4-5	*v*	20.0	+1.25	+15.63
4-6	*e*	26.1	−1.514	−39.52	...	*w*	12.5	......	......
(c) Deflection Point 3									
3-5	*c*	25.0	+1.39	+34.75	4-5	*u*	20.0	+1.25	+25.00
3-4	*d*	32.0	−2.62	−83.84	2-3	*v*	32.5	+1.66	+33.2
2-4	*e*	27.9	−0.857	−23.91	...	*w*	20.0	......	......
(d) Deflection Point 1									
1-3	*c*	25.0	+0.50	+12.5	2-3	*u*	32.5	+1.66	+53.95
1-2	*d*	41.0	−3.02	−123.82	0-1	*v*	50.0	+1.75	+56.88
0-2	*e*	30.5	−0.555	−16.93	...	*w*	32.5	......	......

has been used in computing the changes of length. The deflection points are 1, 3, 5, and 7. The values of Δ are the deflections downward of 1 from 3, 3 from 5, etc. The deflection of Point 0 is obtained by adding the elongation of Member 0-1 to the deflection of Point 1. The horizontal deflection of Point 0 is obtained by applying Equation (5). The calculated deflections downward from Panel Point 9 are transposed to give upward deflections from Point 0.

TABLE 3.—REACTIONS, 200-FOOT ARCH (EXAMPLE 2).

Deflection points (1)	Vertical deflection, Δ, in feet, multiplied by E (2)	Horizontal deflection, H, in feet, multiplied by E (3)	Angular distortion, R (4)	Product Ry, (Column (4) × 50) (5)	Values of n (6)	Values of $Rp\,(n-z)$ at Panel Points: 7 ($R = +0.46$ and $z = 1$) (7)	5 ($R = +0.356$ and $z = 2$) (8)	3 ($R = +0.19$ and $z = 3$) (9)	Summation: $Rpz\,(n-z)$, in feet, multiplied by E (10)	Summation: Vertical deflection, Δ, in feet, multiplied by E (11)	Total Deflection, Δ_T: In feet, multiplied by E (12)	Total Deflection, Δ_T: Transposed (13)	Horizontal reactions, at Point 0 (Column (13) ÷ 99.6) (14)
9	0	....	0	....	0	....	....	....	....	0	0	92.44	0.93
7	+9.54	+2.33	+0.467	+23.35	1	0	....	....	0	+9.54	+9.54	82.90	0.83
5	+10.54	+2.14	+0.356	+17.80	2	+11.68	0	....	+11.68	+20.08	+31.76	60.68	0.61
3	+7.18	+1.39	+0.190	+9.50	3	+23.35	+8.9	0	+32.25	+27.26	+59.51	32.93	0.33
1	+5.85	+0.49	+0.110	+5.50	4	+35.03	+17.8	+4.75	+57.58	+33.11	+90.69	1.75	......
0	+7.50	....	....	....	....	....	....	....	....	+34.86	+92.44	0	......
....	....	+6.35	....	+56.15		$\Sigma\,(H-R_y) = -49.8 \times 2 = -99.6$ for full span							......

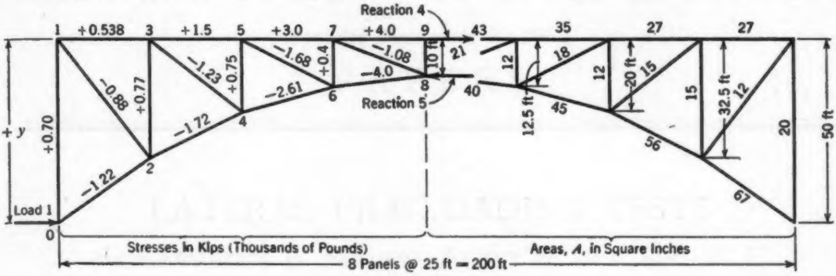

Fig. 5.

Conclusion

The "panel deflection method" solves the problem of truss deflections in a simpler and more direct manner than other analytical methods. It has the advantage of effecting a large saving in the work of computation.

The principal distortions are expressed by means of formulas, which makes the computation easy and accurate. Two short examples have been given to illustrate the practical application of the method. It was not considered necessary to include the extended calculations of a large structure. The writer, however, has applied the method to the solution of most of the problems of indeterminate structures and to the computation of the stresses in a very large structure.

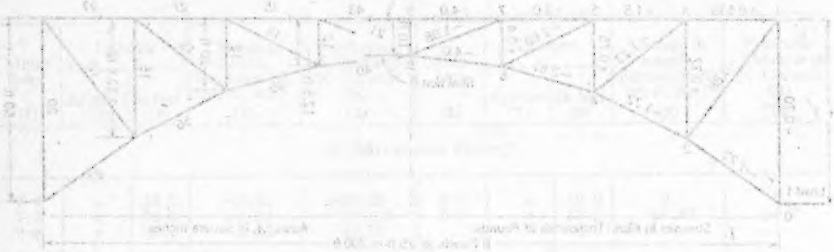

FIG. 5.

CONCLUSION

The "panel deflection method" solves the problem of truss deflections in a simpler and more direct manner than other analytical methods. It has the advantage of effecting a large saving in the work of computation.

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PAPERS

LATERAL PILE-LOADING TESTS

BY LAWRENCE B. FEAGIN[1], ASSOC. M. AM. SOC. C. E.

SYNOPSIS

Tests conducted at Lock and Dam No. 26, Alton, Ill., to determine the resistance under lateral loads of timber and concrete piles, driven in Mississippi River sand, are described in this paper. The work includes descriptions of tests on single piles with heads not fixed, and on groups of four, twelve, and twenty piles with heads fixed in concrete test monoliths. The field data are presented in tabular or graphical form, and the results discussed. In view of the fact that these tests were conducted in only one type of soil, and in view of the many variables involved, no mathematical analysis is included. It is hoped, however, that the results of tests conducted by others may be presented in the discussion.

INTRODUCTION

Lock and Dam No. 26 on the Mississippi River, at Alton, Ill., is the farthest down stream of the twenty-six locks and dams included in the general program of canalization of the Upper Mississippi River. It is situated 23 miles up stream from St. Louis, Mo., 8 miles above the mouth of the Missouri River, and 15 miles below the mouth of the Illinois River. In addition to being a part of the 9-ft canalization of the 650-mile section of river below the "Twin Cities"—St. Paul and Minneapolis, Minn.—the pool above Lock and Dam No. 26 will also form a part of the waterway from the Great Lakes to the Gulf of Mexico.

The designs of twenty-two of the twenty-six locks and dams are entirely, or in part, on piles, driven in most cases in river sand of varying degrees of coarseness. Lock and Dam No. 26 is founded entirely on piles, and its designed lift at extreme low water of 25.2 ft is nearly double that of any of the other projects founded on piles. Under the twin locks there are about 14 200 timber piles and nearly 5 000 concrete piles, and under the dam, about 13 100 timber piles. Although many vertical pile-loading tests have been made in various soils, it appears that comparatively few

NOTE.—Discussion on this paper will be closed in February, 1936, *Proceedings*.

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field tests have been made of the resistance of piles to movement under lateral loads; such, for example, as those resulting from back-fill behind the land-wall of a lock, or from water pressure on dams. It is hoped, however, that lateral loading tests made by others will be fully described in discussion of this paper.

Purpose of Tests

The general purpose of the tests herein described was to secure data on the movement of timber and concrete piles in groups of various sizes when subjected to lateral loads. In as much as the design lateral load assumed under certain conditions, for the piles beneath the river wall of the auxiliary lock, is 6.5 tons per pile (which, in general, may be regarded as rather high), it was particularly desirable, furthermore, to determine the degree of safety of this assumption. Information was also desired for use in designing the foundations for the piers of Dam No. 26, and for designing future locks and dams on similar foundations.

Test Monoliths

In order to simulate, as nearly as practicable, the actual loading conditions that will occur in the completed structures, six concrete monoliths were constructed on timber and concrete piles driven along the toe of the slope of the Illinois shore behind the land-wall of the main lock, as shown in Figs. 1 and 2. The heads of the piles were fixed, as in the lock-walls,

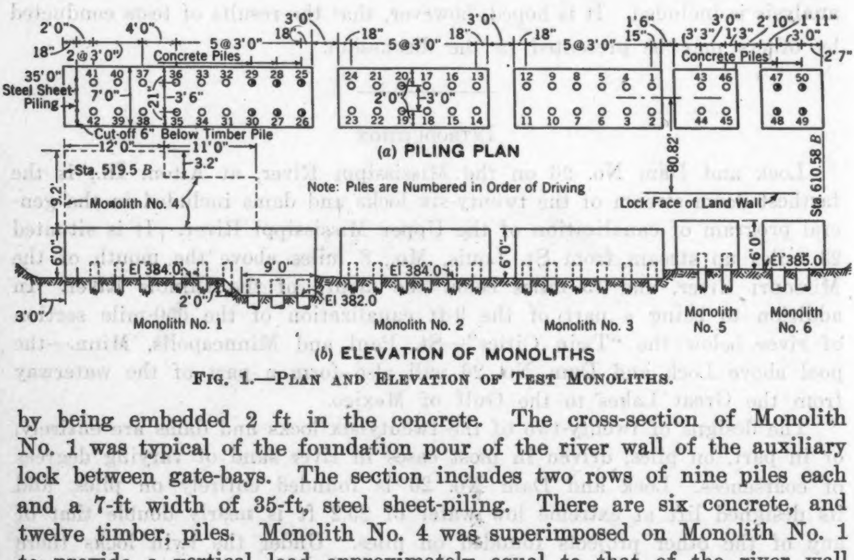


Fig. 1.—Plan and Elevation of Test Monoliths.

by being embedded 2 ft in the concrete. The cross-section of Monolith No. 1 was typical of the foundation pour of the river wall of the auxiliary lock between gate-bays. The section includes two rows of nine piles each and a 7-ft width of 35-ft, steel sheet-piling. There are six concrete, and twelve timber, piles. Monolith No. 4 was superimposed on Monolith No. 1 to provide a vertical load approximately equal to that of the river wall under the eccentric loading that occurs when the auxiliary lock is filled and the river below the dam is at extreme low water. A comparison of the loading conditions is shown in Fig. 3. Monoliths Nos. 2 and 3 are each on two rows of six timber piles each, or twelve piles to the monolith.

FIG. 2.—VIEW OF TEST MONOLITHS, UPPER MISSISSIPPI RIVER, LOCK NO. 26.

Monolith No. 5 is supported by four timber piles and Monolith No. 6 by four concrete piles. Thus, provision was made for comparing the movement

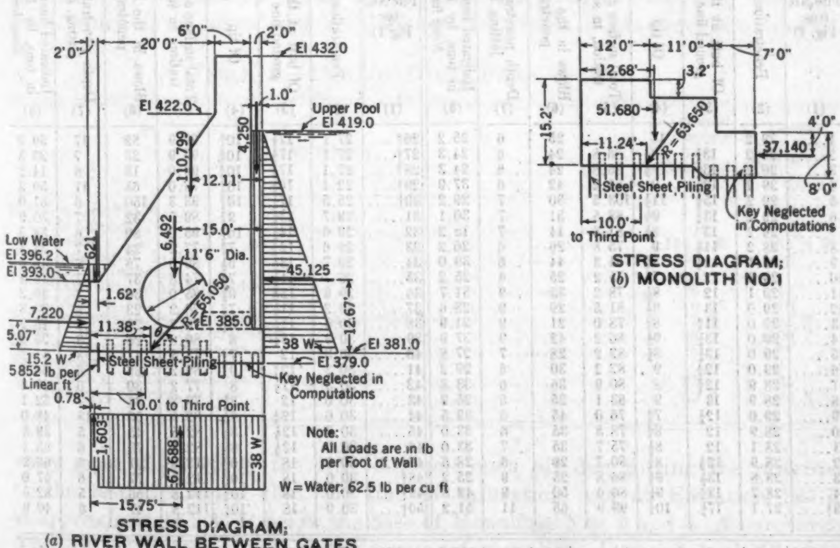


FIG. 3.—LOAD AND STRESS DIAGRAMS.

of monoliths on groups of four, twelve, and twenty piles, assuming the steel sheet-piling under Monolith No. 1 as equivalent to two piles. In addition to supplying the means of making lateral load tests, the monoliths were used for making studies of relative volume changes, heats of hydration, etc., with various types of cement and methods of curing.

Single Piles

In order to compare the movement of groups of piles with heads fixed in the concrete to that of a single pile with head not fixed, lateral tests were also made on a single timber pile and on a concrete pile driven as a part of the foundation of the land-wall of the locks.

Driving and Physical Data

Table 1 contains pile-driving and physical data for each pile under the test monoliths, and Fig. 4, supported by Table 2, shows corresponding data for the single concrete and timber piles tested.

Foundation Conditions

A log of a hole, *O,* bored in the vicinity of Monolith No. 1, is shown in Fig. 5. It will be noted that the foundation consists of medium sand.

TABLE 1.—Pile-Driving Data*

Pile No. (see Fig. 1)	Penetration, in feet	Diameter, in Inches: Of butt at the ground line	Diameter, in Inches: Of tip	Total surface area of penetration, in square feet	Blows in the last foot of penetration	Depth penetrated without jetting, in feet	Indicated bearing value, in tons, by *Engineering News* formula	Pile No. (see Fig 1)	Penetration, in feet	Diameter, in Inches: Of butt at the ground line	Diameter, in Inches: Of tip	Total surface area of penetration, in square feet	Blows in the last foot of penetration	Depth penetrated without jetting, in feet	Indicated bearing value, in tons, by *Engineering News* formula
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1.....	29.2	....	11	....	25	6	25.2	26†....	27.1	17½	10¾	99.9	82	17	59.2
2.....	29.2	13¼	9½	86.8	24	6	24.3	27†....	27.1	17½	10¾	99.9	23	7	23.8
3.....	29.2	13¼	9½	86.8	24	6	24.3	28†....	27.1	17½	10¾	99.0	13	8	14.2
4.....	29.2	12¼	9	81.2	42	6	37.9	29†....	22.1	16½	10¾	78.0	63	11	50.2
5.....	29.2	15½	11½	103.5	30	7	29.2	30†....	25.5	17¼	10¾	93.3	150	6	81.0
6.....	29.2	13¾	9½	88.5	31	7	30.1	31.....	29.7	13¼	9¾	89.5	32	7	30.9
7.....	29.2	13	8¼	81.6	14	7	15.2	32.....	29.6	14¼	10¼	95.0	80	6	58.2
8.....	28.2	11½	9	75.8	26	6	26.2	33.....	29.6	12½	7½	77.2	24	7	24.3
9.....	29.1	12½	8¾	81.3	44	6	39.0	34.....	29.7	13½	9½	89.5	76	7	56.5
10.....	29.3	12	7¾	75.2	25	6	25.2	35.....	29.7	11½	7¾	74.8	37	7	34.5
11.....	29.1	12	8½	78.2	33	9	31.7	36.....	29.6	12½	9½	85.0	30	7	29.2
12.....	29.0	13	8½	81.5	29	9	28.6	37.....	30.2	11½	7¾	76.2	23	8	23.8
13.....	29.0	11¾	8¾	78.0	21	9	21.9	38	30.5	11½	8¼	78.7	20	7	20.9
14.....	29.0	13¼	9½	86.2	42	9	37.9	39.....	30.3	12¼	8	80.0	25	8	25.2
15.....	29.0	13¼	8½	82.2	28	7	27.8	40.....	30.3	12	7¾	78.3	41	8	37.1
16.....	29.0	12¾	9	82.2	30	6	29.2	41.....	30.2	12¾	9¼	89.2	26	6	26.1
17.....	28.9	12¾	8	80.9	36	6	33.8	42.....	30.3	11½	8	77.2	30	6	29.2
18.....	28.9	13	9	83.1	25	6	25.2	43.....	30.0	12	8¼	79.3	34	7	32.1
19.....	29.0	12¼	7¾	76.0	45	6	39.5	44.....	30.0	12¾	8½	82.5	59	13	48.0
20.....	28.9	12	8¾	78.5	35	6	33.0	45.....	30.0	12½	8½	82.5	45	5	39.5
21.....	28.1	12	8½	75.7	35	7	33.0	46.....	30.0	12½	8½	82.5	73.	6	55.1
22.....	28.9	12½	8¾	80.7	29	6	28.5	47†....	30.0	18	10¾	112.8	101	6	66.9
23.....	28.8	13½	9½	86.8	25	9	25.2	48†....	30.0	18	10¾	112.8	77	6	57.0
24.....	28.7	13¾	9¼	86.6	50	9	42.8	49†....	30.0	18	10¾	112.8	156	5	82.5
25†....	27.1	17½	10¾	99.9	65	11	51.2	50†....	30.0	18	10¾	112.8	47	5	40.9

* Measured stroke of ram, 35 in.; weight of ram, 5 000 lb; and jet pressure, 100 lb. All timber piles were oak, principally white oak.

† Concrete pile.

The samples, of which sieve analyses are given in Fig. 5, were taken by jetting a 2-in. pipe about 3 ft below a 3-in. casing, stopping the jet, and permitting sand to enter the 2-in. pipe through side openings, and then

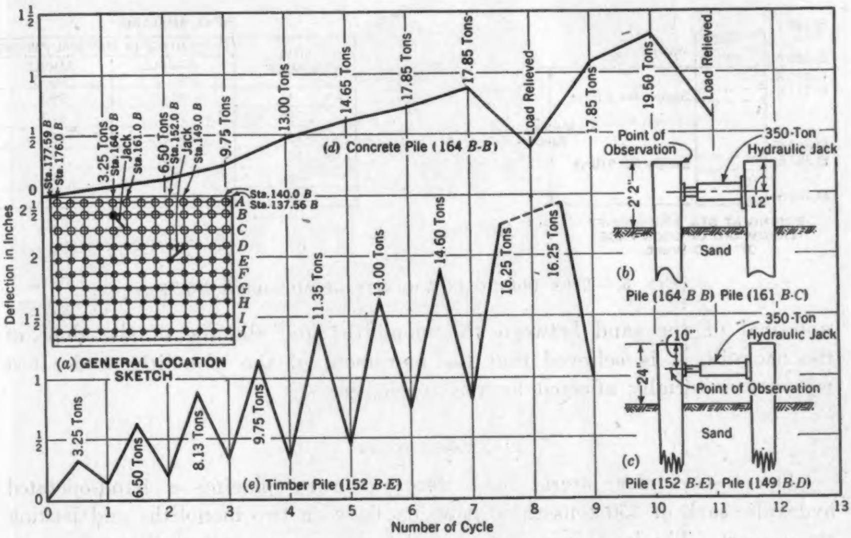


FIG. 4.—DEFLECTIONS OF A SINGLE CONCRETE PILE.

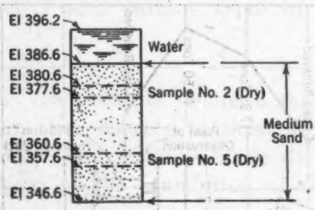
withdrawing the pipe. Although this method affords a reasonably representative sample, the sample tends to be somewhat coarser than the material from which it is taken, due to the fact that some of the fines are washed

TABLE 2.—DRIVING DATA, PILES IN FIG. 4

Description (1)	Tapered concrete pile (Fig. 4 (*a*)) (2)	Timber pile (oak) (Fig. 4 (*b*)) (3)
Location (see Fig. 4 (*a*)):		
Pile number	*B*	*E*
Station	164.0 *B*	152.0 *B*
Total penetration, in feet	30	30
Diameter, in Inches:		
Of the butt, at the ground line	18	14
Of the tip	10.75	10
Total surface area, in square feet, below the ground line	112.8	94.2
Blows in the last foot of penetration	68	100
Penetration without using a jet	5	7
Engineering News Formula:		
Length of stroke, in inches	35	35
Indicated bearing, in tons	52.7	66.3

away prior to stopping the jet. A small open pit dug during the progress of the tests indicated that the line of saturation was at Elevation 381.7, or approximately 2.33 ft below the base of Monoliths Nos. 2 and 3. A surcharge was created at each end of the group of monoliths by sand back-fill. At the up-stream end of Monolith No. 1, however, the back-fill above its base

was removed for a distance of 3 ft, and at the down-stream end of Monolith No. 6, approximately 5 ft. In view of the small movement of the monoliths during the tests, and the fact that there was no perceptible



BORING AT STA. 500 *B*; 68 FT LANDWARD OF LOCK FACE OF LAND WALL

SIEVE ANALYSIS

SIEVE NUMBER	PERCENTAGE OF MATERIAL PASSING	
	SAMPLE NO. 2	SAMPLE NO. 5
4	98.7	99.3
8	97.1	97.5
16	89.2	68.1
28	56.5	7.7
48	3.2	0.2
100	0.1	0.0

FIG. 5.—TEST BORING IN VICINITY OF MONOLITH NO. 1.

upheaval of the sand between the monoliths and the toe of the slope of the back-fill, it is believed that the movement of the monoliths under test was not materially affected by the surcharge.

EQUIPMENT

The tests under lateral load were made by placing a hand-operated hydraulic jack of 350 tons rated capacity between two monoliths and jacking them apart. The loads were determined from an accurately calibrated gauge. The jacking equipment is shown in Figs. 6 and 7. In order to secure sufficient resistance to move Monolith No. 1, heavy oak struts were placed between Monoliths Nos. 2, 3, 5, and 6. The point of application of the lateral load was 2.5 ft above the base of the monoliths for the test on Monoliths Nos. 5 and 6, and 2.0 ft above the base for the tests on Monoliths Nos. 1, 2, and 3. Thus, the point of application was at the general level of the tops of the piles.

OBSERVATIONS

Observations were taken by reading with a transit a rule held over the jack with its end against a point on the center line of the monolith and 2.25 ft below the top. Readings were taken to the nearest $\frac{1}{32}$ in. Additional observations were taken at the top and sides of each monolith to determine whether or not there was twisting or tilting of the monoliths. In general, however, it was found that there was no appreciable twisting or tilting. The movement of the single piles in the foundation of the land-wall was taken by jacking two piles apart and observing their spread.

SEQUENCE OF TESTS

The first tests, which were somewhat of a preliminary nature, were made on the single timber and concrete piles of the foundation of the land-wall, with butts not fixed. Details are shown in Figs. 4 (*b*) and 4 (*c*). The first test on the monoliths (see Fig. 8 (*a*)) was made by jacking

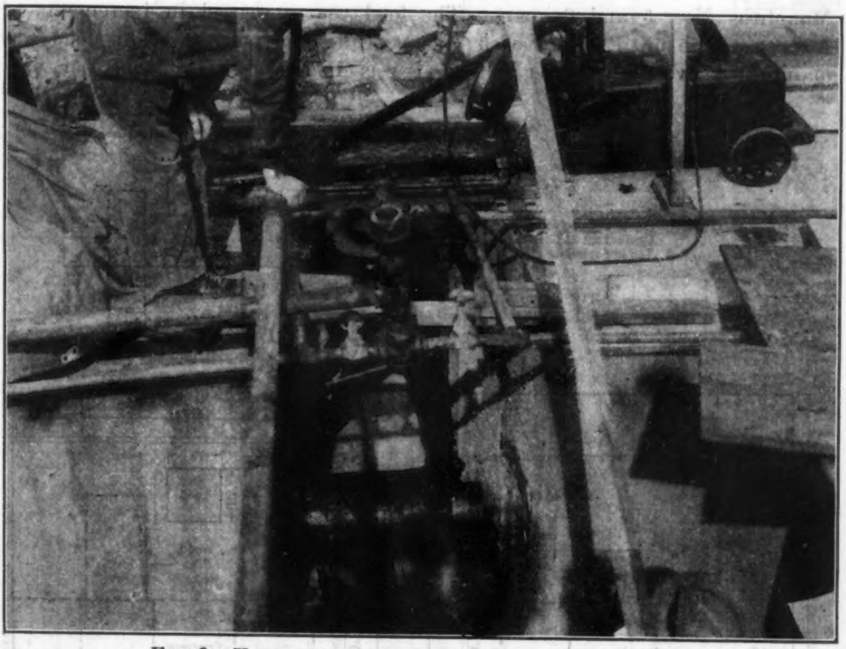


FIG. 6.—HORIZONTAL LOADING TEST BETWEEN MONOLITHS.

FIG. 7.—LATERAL LOADING TEST BETWEEN TIMBER FOUNDATION PILES, (152 *B-E*) AND (149 *B-D*). (SEE FIG. 4(*a*)).

Monoliths Nos. 5 and 6 apart. The load was increased in increments of 1 ton per pile until a load of 5 tons per pile was reached. This load was maintained overnight, after which it was increased to 30 tons per pile.

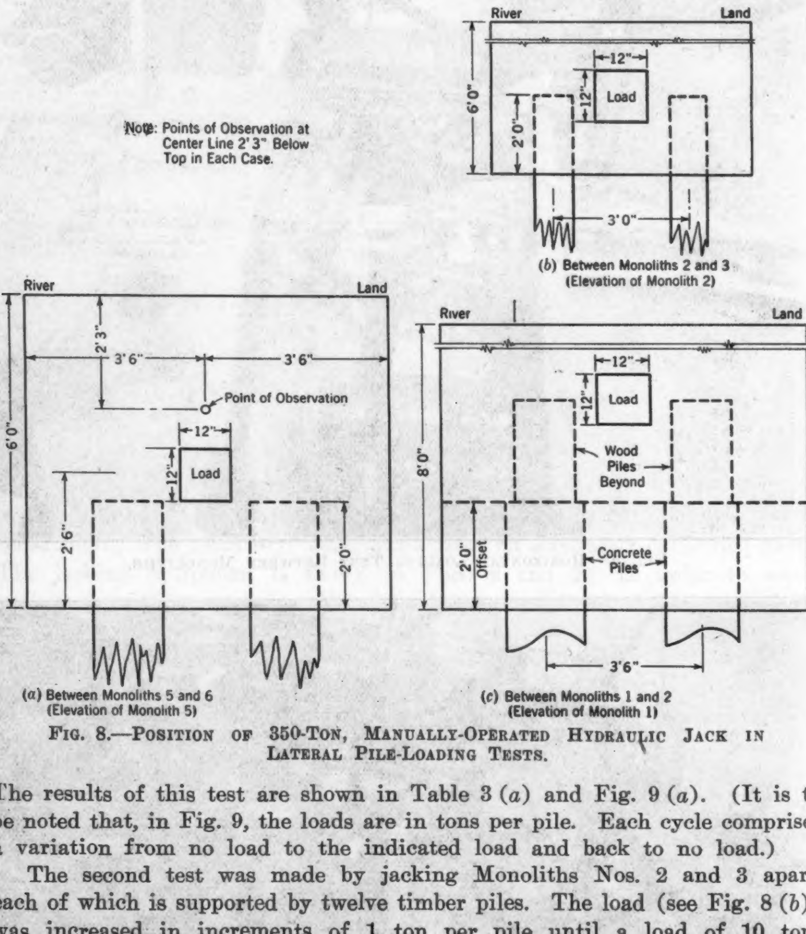


FIG. 8.—POSITION OF 350-TON, MANUALLY-OPERATED HYDRAULIC JACK IN LATERAL PILE-LOADING TESTS.

The results of this test are shown in Table 3 (*a*) and Fig. 9 (*a*). (It is to be noted that, in Fig. 9, the loads are in tons per pile. Each cycle comprises a variation from no load to the indicated load and back to no load.)

The second test was made by jacking Monoliths Nos. 2 and 3 apart, each of which is supported by twelve timber piles. The load (see Fig. 8 (*b*)) was increased in increments of 1 ton per pile until a load of 10 tons per pile was reached, which was then maintained overnight, after which it was increased to 20 tons per pile. Following each increment the load was released and the permanent deflection observed. The results of this test are shown in Table 3 (*b*) and Fig. 9 (*b*).

The third test was made by placing the jack between Monoliths Nos. 1 and 2 (see Fig. 8 (*c*)) and placing oak struts between Monoliths Nos. 2, 3, 5, and 6. A load of 4 tons per pile, assuming the steel sheet-piling as the equivalent of two piles, was maintained overnight. This load was then increased to 6.5 tons per pile (the design load of the river wall of the auxiliary lock) and maintained over the week-end. The load was then

TABLE 3.—Lateral Pile-Loading Tests

Date: September (1)	Time (2)	Load, in Tons: Total (3)	Load, in Tons: Per pile (4)	Deflection in Thirty-Secondths of an Inch, A: Deflection reading (5)	A: Permanent deflection¶ (6)	B: Deflection reading (7)	B: Permanent deflection¶ (8)
(a) A = Monolith No. 5; B = Monolith No. 6							
26.....	5:15 P.M.	0	0	0	0	0	0
26.....	....	4	1.0	0	0	0	0
26.....	....	8	2.0	0	0	0	0
26.....	....	12	3.0	1	0	0	0
26.....	....	12	3.0	1	0	0	0
26.....	....	12	3.0	1	0	0	0
26.....	....	16	4.0	2	0	1	0
26.....	....	16	4.0	2	1	1	1
26.....	....	16	4.0	2	1	1	....
26.....	5:30 P.M.	20	5.0	3	1	2	....
27.....	9:15 A.M.*	20	5.0	4	1	3	2
27.....	....	20	5.0	4	2	3	1
27.....	....	20	5.0	4	2	3	2
27.....	....	20	5.0	5	2	3	2
27.....	....	24	6.0	5	2	4	2
27.....	....	26	6.5	6	2	4	2
27.....	....	26	6.5	6	3	4	2
27.....	....	26	6.5	7	3	4	2
27.....	....	26	6.5	7	3	4	2
27.....	....	28	7.0	7	3	5	2
27.....	....	28	7.0	8	4	5	3
27.....	....	28	7.0	8	4	5	3
27.....	....	32	8.0	9	4	5	3
27.....	....	36	9.0	10	5	7	3
27.....	....	40	10.0	12	6	8	4
27.....	....	48	12.0	14	....	10	....
27.....	....	56	14.0	17	....	13	....
27.....	....	64	16.0	20	9	16	9
27.....	....	80	20.0	29	14	26	14
27.....	....	100	25.0	42	....	42	....
27.....	11:00 A.M.	120	30.0	56	30	56	....
(b) A = Monolith No. 2; B = Monolith No. 3							
27.....	2:00 P.M.	24	2.0	0	....	1	....
27.....	....	36	3.0	1	0	2	0
27.....	....	48	4.0	2	1	4	1
27.....	....	60	5.0	3	2	4	2
27.....	....	60	5.0	4	2	6	2
27.....	....	60	5.0	4	....	6	....
27.....	....	72	6.0	5	2	6	2
27.....	....	72	6.0	6	2	7	3
27.....	....	72	6.0	6	3	8	3
27.....	....	72	6.0	6	....	7	....
27.....	....	78	6.5	6	3	8	3
27.....	....	78	6.5	7	3	9	5
27.....	....	78	6.5	7	....	9	....
27.....	....	84	7.0	8	4	9	4
27.....	3:00 P.M.	84	7.0	8	4	9	4
27.....	....	96	8.0	9	4	11	5
27.....	....	96	8.0	9	5	11	6
27.....	....	96	8.0	10	....	12	....
27.....	....	108	9.0	10	5	13	7
27.....	....	108	9.0	11	5	14	7
27.....	....	108	9.0	11	....	16	....
27.....	....	120	10.0	11	6	16	8
27.....	....	120	10.0	12	6	16	9
27.....	3:05 P.M.	120	10.0	12	6	16	10
27.....	4:05 P.M.	120	10.0	12	....	16	....
28.....	9:40 A.M.	120	10.0	14	8	17	10
28.....	....	120	10.0	14	....	17	....
28.....	....	132	11.0	15	8	18	10
28.....	....	132	11.0	15	....	18	....
28.....	....	144	12.0	16	8	20	10
28.....	....	144	12.0	17	9	22	11
28.....	....	144	12.0	17	....	21	....
28.....	....	168	14.0	20	10	26	12
28.....	....	168	14.0	21	....	25	....
28.....	....	192	16.0	26	13	28	14
28.....	....	192	16.0	30	....	30	....
28.....	....	192	16.0	30	....	30	....
28.....	10:25 A.M.	240	20.0	38	18	39	20

Date: September and October (1)	Time (2)	Load, in Tons: Total (3)	Load, in Tons: Per pile (4)	Deflection, in Thirty-Secondths of an Inch, A: Deflection reading 5	A: Permanent deflection¶ 6
(c) A = Test Monolith No. 1					
28	1:30 P.M.	0	0	0	0
28	....	80	4.0	2	0
28	....	80	4.0	2	0
28	1:50 P.M.	80	4.0	2	....
29	9:15 A.M.	80	4.0	4	2
29	....	100	5.0	4	2
29	....	120	6.0	4	2
29	9:45 A.M.	130	6.5	5	....
1	10:15 A.M.	130	6.5	5	2
1	....	130	6.5	5	2
1	....	130	6.5	5	2
1	....	130	6.5	5	2
1	....	130	6.5	5	2
1	....	130	6.5	5	3
1	....	130	6.5	5	3
1	....	130	6.5	5	3
1	....	130	6.5	5	3
1	....	130	6.5	5	3
1	....	130	6.5	5	3
1	....	130	6.5	5	3
1	....	130	6.5	5	3
1	....	130	6.5	5	3
1	....	130	6.5	5	3
1	....	130	6.5	5	3
1	....	130	6.5	5	3
1	....	130	6.5	7	3
1	....	130	6.5	7	3
1	....	130	6.5	7	3
1	....	130	6.5	7	3
1	....	130	6.5	7	3
1	....	130	6.5	7	3
1	....	130	6.5	7	3
1	....	130	6.5	7	4
1	....	130	6.5	7	4
1	10:50 A.M.	130	6.5	7	4
1	1:10 P.M.	130	6.5	7	4
1	....	130	6.5	8	4
1	....	130	6.5	8	4†
1	....	130	6.5	8	4†
1	....	130	6.5	8	4†
1	....	130	6.5	8	4†
1	....	130	6.5	8	4†
1	1:50 P.M.	130	6.5	8	4
1	3:00 P.M.	130	....	....	4
1	....	140	7.0	8	4
1	....	160	8.0	8	4
1	....	180	9.0	9	4
1	....	200	10.0	11	4
1	3:45 P.M.	200	10.0	12	‡
2	11:00 A.M.	200	10.0	15	6
2	12:00 A.M.	200	10.0	20	‡
4	9:20 A.M.	240	12.0	26	16
4	....	280	14.0	31	18
4	....	320	16.0	36	....
4	10:30 A.M.	366	18.3	45	26

* Load of 5 tons per pile sustained overnight. † Five cycles of load. ‡ 25 cycles of load.
¶ Permanent deflection is deflection observed after indicated load was removed.

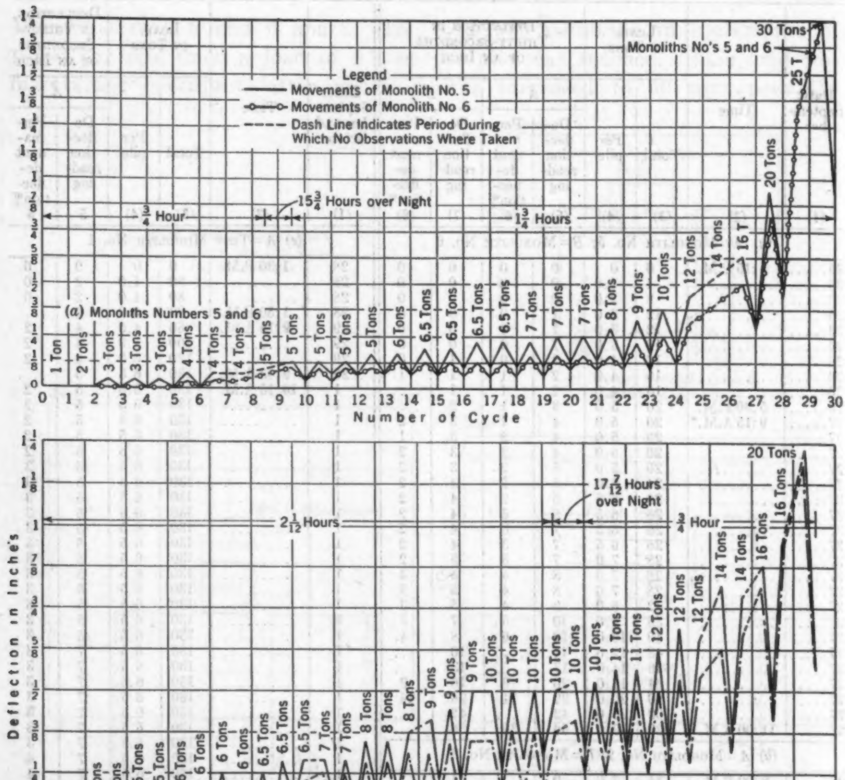


FIG. 9.—DEFLECTION OF TEST MONOLITHS.

alternated from 0 to 6.5 tons for 53 cycles, after which it was increased to 10 tons and alternated from 0 to 10 tons for 25 cycles. It was then maintained at 10 tons per pile overnight and again alternated from 0 to 10 tons for 25 cycles. After continuing to maintain a load of 10 tons for two more days, the load was then increased to a total of 366 tons, or 18.3 tons per pile—the capacity of the jack—and the test was completed. From the morning of October 1 to the completion of the test on October 4, the soil around Monolith No. 1 was thoroughly saturated by a hose from which water was allowed to run continuously. The results of this test are shown in Table 3 (*c*) and Fig. 9 (*c*).

Discussion of Results

A comparison of the deflection of the single piles and the monoliths for different loads is given in Fig. 10 (*a*). It will be noted that the least deflection was shown by Monolith No. 6, on four concrete piles (see Fig. 1). That of Monolith No. 5 on four timber piles was greater than that of Monolith No. 6 by a maximum of $\frac{1}{8}$ in. at 10 tons, and, for loads from 25 to 30 tons per pile, the movement of the two monoliths was the same. The maximum deflection for any of the monoliths at 4 tons was $\frac{1}{8}$ in. and at 6.5 tons, $\frac{9}{32}$ in. For all practical purposes it appears that for loads less than about 6 tons per pile the deflection of all monoliths was about the same.

It will be noted by referring to Fig. 9 that sustained loads and repetitions of load resulted in progressive deflection. In the case of Monolith No. 1, 53 cycles of load from 0 to 6.5 tons resulted in an increase in deflection from $\frac{5}{32}$ in. to $\frac{1}{4}$ in., and 53 cycles from 0 to 10 tons resulted in an increase from $\frac{15}{32}$ in. to $\frac{13}{16}$ in. The effect of sustaining the load appeared to be less than frequent repetitions of the same load; for example, applying a sustained load of 6.5 tons for 48 hr on Monolith No. 1 resulted in no increase in deflection, whereas 53 cycles from 0 to 6.5 tons added $\frac{3}{32}$ in. to the deflection. The greater deflection of Monolith No. 1 may have been due, in large measure, to the fact that it was subjected to 128 cycles, whereas the other monoliths underwent less than 40 cycles.

Level readings taken on a point fixed in the top of Monolith No. 1 indicated that no vertical movement occurred during the test on that monolith. There was a permanent deflection or set for all monoliths for loads of 4 tons, or more. For loads of 6.5 tons this permanent set amounted to from $\frac{1}{16}$ in. to $\frac{1}{8}$ in. For maximum loads of 30 tons per pile the maximum deflection of Monoliths Nos. 5 and 6 was $1\frac{3}{4}$ in. and the permanent deflection, or set, was $1\frac{5}{8}$ in. Even under such severe loading there was no definite failure, and subsequent examination did not disclose any distress in the fibers of the piles.

Observations taken on the up-stream end of Monolith No. 1 near the base and at the top of Monolith No. 4 indicated that there was no tilting. Tilting would not be expected, however, unless the vertical loads on the piles at the toe caused settlement, or unless the lateral loads were such that the piles at the heel might be pulled upward, or both. An examination

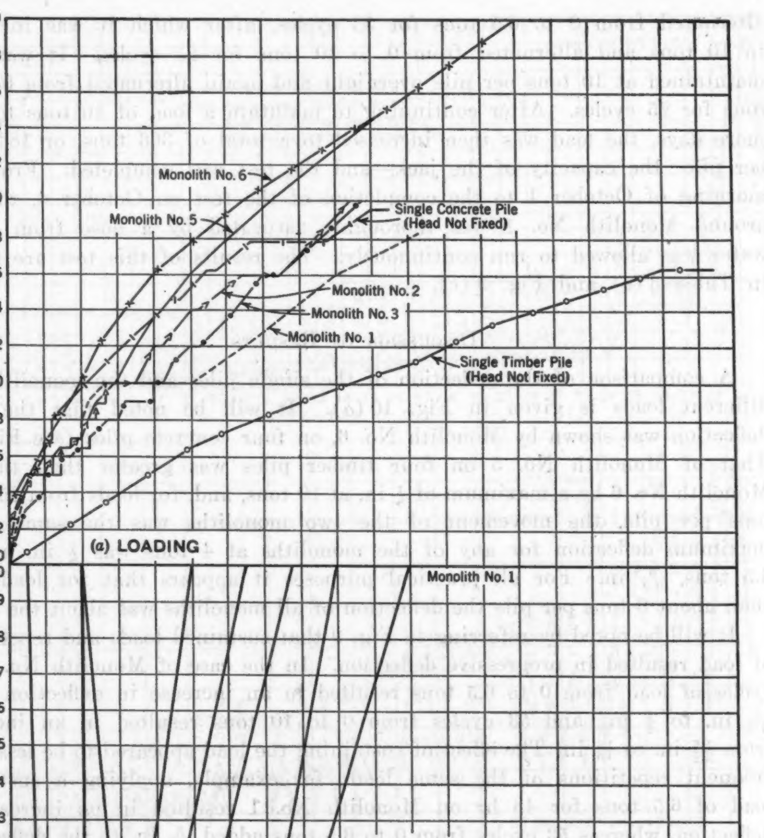


FIG. 10.—EFFECT OF LOADING AND NUMBER OF PILES, ON THE LATERAL DEFLECTION OF A FOOTING.

of the sizes of the piles indicates that the average shear for loads of 6.5 tons per pile was about 85 lb per sq in., which is not serious.

In order to determine the effect of the size of the group of piles loaded, the movement of each monolith was plotted against the number of piles in the group (see Fig. 10 (*b*)). For loads of less than about 6.5 tons per pile the size of the group appears to have no influence on the deflection, whereas, for higher loads, the deflection appears to be less for the smaller group.

Fiber Stress Investigation

Following the completion of the foregoing tests the sand was removed beneath the up-stream end of Monolith No. 2 to a depth of 2.17 ft, the level of the line of saturation, thereby exposing two of the timber piles. A careful examination of these piles failed to disclose any indication of failure in the fibers. Plugs were then inserted on 10-in. centers in the up-stream and down-stream sides of the two up-stream piles. The top plugs were 8 in. below the base of the monolith (see Fig. 11). By means of an extensometer,

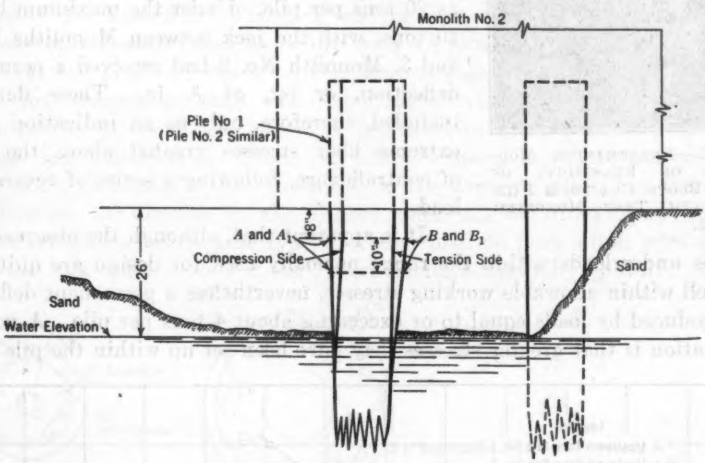


Fig. 11.

observations were then taken on the strain in the outside fibers of the piles both on the up-stream, or compression, side and on the down-stream, or tension, side, for various lateral loads ranging from 6.5 to 30 tons per pile.

In order to determine the modulus of elasticity of the oak piles and thereby determine the fiber stress, the two piles were sawed off beneath the monolith, and blocks were tested in the compression machine in the Concrete Laboratory. The strain was determined by means of a compressometer as shown in Fig. 12. In Fig. 13 (*b*) is shown the derivation of the modulus of elasticity, which was determined as $\frac{432}{0.000230} = 1\,878\,260$, and in Fig. 13 (*a*), the fiber stress in bending for various loads as determined by the extensometer measurements both for the compression and tension side of the piles. It will be noted that the fiber stress in the compression side

was only about 60% of that in the tension side. This would seem to indicate that there was a tendency to pull the pile upward which partly counterbalanced the compression from bending in the compression side and increased the tension on the tension side. It is probable that the strain measured may have been affected by the fact that the plugs were inserted between the point of fixation of the head of the pile and the point of contraflexure. The data shown in Fig. 13 are based on observations repeated with great care and are believed to be essentially correct. Prior to the time that they were taken, however, Monolith No. 2 had been subjected to lateral loads from both directions during the tests outlined previously herein (under the heading "Sequence of Tests") with loads as great as 20 tons per pile. Under the maximum load of 20 tons, with the jack between Monoliths Nos. 2 and 3, Monolith No. 2 had received a permanent deflection, or set, of $\frac{9}{16}$ in. These data are included, therefore, only as an indication of the extreme fiber stresses created above the point of contraflexure, following a series of reversals of load.

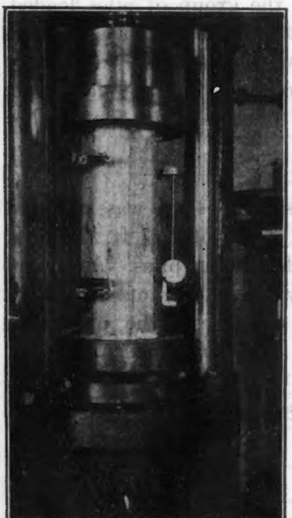


FIG. 12.—DETERMINING MODULUS OF ELASTICITY OF OAK BLOCK CUT FROM PILE BENEATH TEST MONOLITH NO. 2.

It is apparent that, although the observed fiber stresses under loads within the range normally used for design are quite low, and well within allowable working stresses, nevertheless a permanent deflection was produced by loads equal to or exceeding about 4 tons per pile. A possible explanation is that greater stresses may have been set up within the pile below

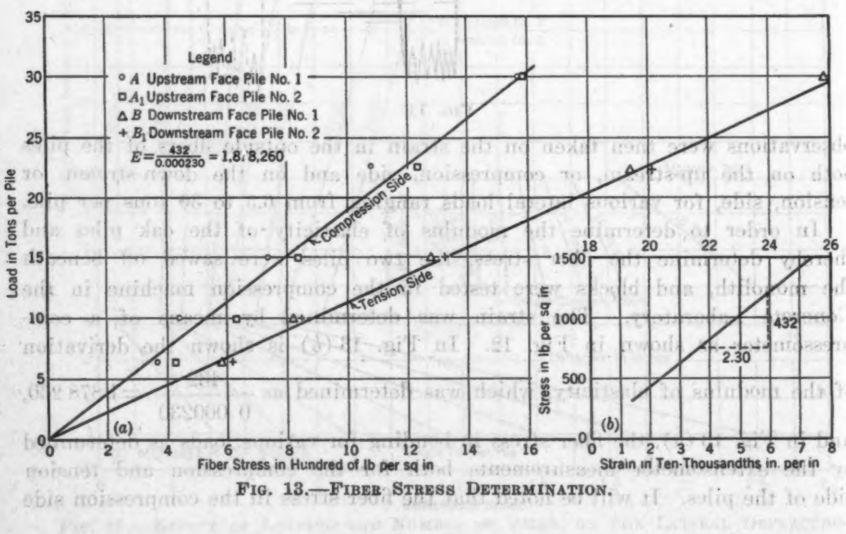


FIG. 13.—FIBER STRESS DETERMINATION.

the point of contraflexure not only in bending but in direct tension as a result of an upward force tending to pull up the pile. The point at which the elastic limit is reached is somewhat conjectural, as lateral movement of the pile may be accompanied by a displacement of the sand which might fill the space left on the back side of the pile, thereby preventing its full return to its original position. An examination of these piles at the base of the concrete showed that under load the bond between the pile and the concrete was broken on the down-stream, or tension, side of the pile, forming a crack in which the blade of a pocket-knife could be inserted. Nevertheless, the head of the pile was still firmly held in the concrete. The sand was also dug out to the water-table beneath a part of Monoliths Nos. 5 and 6 after the foregoing tests. A careful examination of both concrete and timber piles showed no indication of distress, although these piles had been subjected to a lateral load of 30 tons per pile.

Plugs were then inserted, on 10-in. centers, in the up-stream and down-stream sides of one of the concrete piles, with the top plugs 8 in. below the base of the monolith. The jack was again set between these monoliths and observations were taken as before with an extensometer. Again, it was found that the observed strain on the down-stream or tension side exceeded that in the compression side. The resulting computed stress did not reach 600 lb. per sq in. until the load was increased to 28.5 tons per pile. The load was then increased to 40 tons per pile. Between 30 and 40 tons per pile several circumferential cracks developed in the concrete piles, but there was no indication of distress in the timber piles, all of which were of oak. The concrete piles, which were typical of the concrete foundation piles driven under the lock walls (see Fig. 14), were manufactured by spinning a very

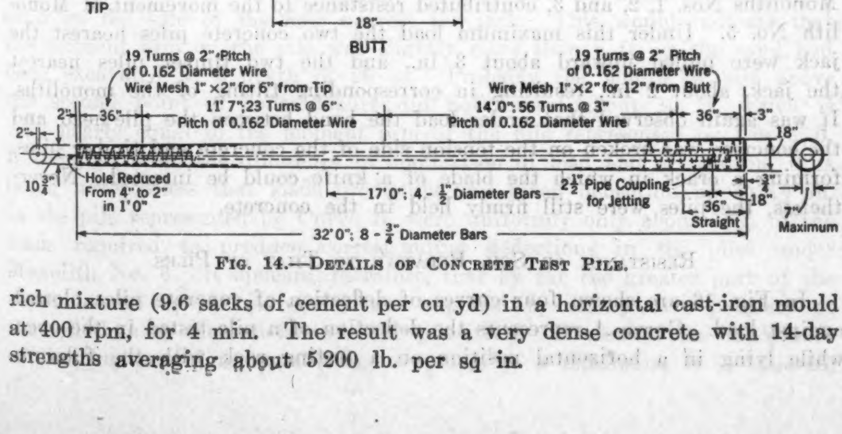


Fig. 14.—Details of Concrete Test Pile.

rich mixture (9.6 sacks of cement per cu yd) in a horizontal cast-iron mould at 400 rpm, for 4 min. The result was a very dense concrete with 14-day strengths averaging about 5 200 lb. per sq in.

The movement of the monoliths was so rapid at the higher load that the resistance was insufficient to permit the building up of a load greater than 40 tons per pile. When the ram of the jack was fully extended, Monolith No. 6 had moved a total of 10 in. and Monolith No. 5, a total of 8 in., measured at a point 2 ft 3 in. below the top of the monoliths (see Fig. 15). In view of the tilting, the corresponding horizontal movement

FIG. 15.—TILTING OF TEST MONOLITHS NOS. 5 AT LEFT, AND NO. 6 AT RIGHT, UNDER LATERAL LOAD OF 40 TONS PER PILE, WITH RAM OF JACK FULLY EXTENDED.

of Monolith No. 6 at the base was approximately 6 in., and that of Monolith No. 5 about 5 in. This difference in movement of the two monoliths may be accounted for, at least partly, by the probability that the piles beneath Monoliths Nos. 1, 2, and 3, contributed resistance to the movement of Monolith No. 5. Under this maximum load the two concrete piles nearest the jack were pulled upward about 3 in., and the two timber piles nearest the jack, about 2 in., resulting in corresponding tilting of the monoliths. It was again observed that under load the bond between the pile-head and the concrete was broken on the tension side of the concrete and timber piles, forming a crack in which the blade of a knife could be inserted. Nevertheless, the piles were still firmly held in the concrete.

RESISTANCE OF SOIL RELATIVE TO THAT OF PILES

In Fig. 16 are shown four curves of deflection of concrete piles plotted against load. Curve *A* represents the deflection of a pile tested in the open while lying in a horizontal position on a testing rack with the fulcrum

at the mid-point and the load at the butt, but with the butt not fixed. Curve *B* is similar to Curve *A*, with the exception that the fulcrum is at the third point. The other two curves represent the deflection of piles

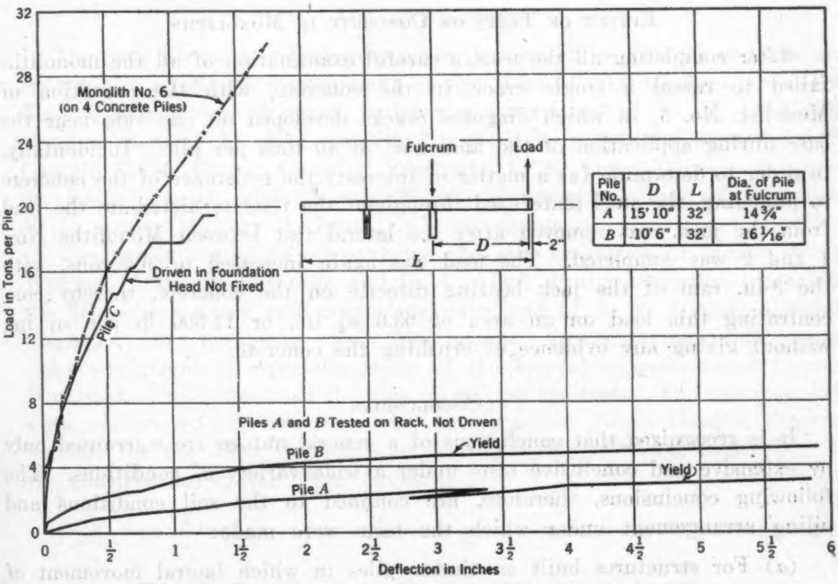


FIG. 16.—DEFLECTIONS FOR VARIOUS FULCRUM LOCATIONS.

driven in the foundation sand, one with head not fixed and the other with heads fixed in Test-Monolith No. 6. All these piles were of the same size and were similarly reinforced. It will be noted, as might be expected, that the deflection at the yield point decreased quite rapidly as the moment arm was decreased.

When a load of approximately 30 tons per pile was being applied during the final test, it was found that a carpenter's rule could be inserted to a depth of 6 ft in the space between the pile and the adjacent foundation sand on the side of the hole nearest the jack. This would indicate that the moment arm on the pile was slightly more than 8 ft, as the sand had been excavated to a depth of 26 in. Probably the length of pile above the second point of contraflexure did not exceed about 10 ft, which is very nearly equal to the moment arm of the pile represented by Curve *B*. Bearing in mind the difference in application of load and of fixation, it is noted nevertheless that loads producing any given deflection up to $1\frac{3}{4}$ in. in the pile represented by Curve *B*, were uniformly only about 14% of the loads required to produce corresponding deflections in the piles under Monolith No. 6. It appears, therefore, that by far the greater part of the resistance to lateral movement was supplied by the passive pressure of the soil than by the resistance of the pile itself. In view of the importance of limiting lateral movements in structures to a minimum, it is believed,

nevertheless, that the structural rigidity of the pile itself is of primary concern, especially for about 10 ft below the base of the concrete foundation, as the line of support of the soil lies several feet below the top of the soil.

Effect of Tests on Concrete in Monoliths

After completing all the tests a careful examination of all the monoliths failed to reveal a single crack in the concrete, with the exception of Monolith No. 5, in which diagonal cracks developed on one side near the base during application of the final load of 40 tons per pile. Incidentally, in order to determine (as a matter of interest) the resistance of the concrete to punching, the steel plate used throughout the tests to distribute the load from the jack, was removed after the lateral test between Monoliths Nos. 1 and 2 was completed. The load was again increased to 366 tons, with the 9-in. ram of the jack bearing directly on the concrete, thereby concentrating this load on an area of 63.6 sq in., or 11 500 lb per sq in., without giving any evidence of crushing the concrete.

Conclusions

It is recognized that conclusions of a general nature are warranted only by extensive and conclusive tests under a wide variety of conditions. The following conclusions, therefore, are confined to the soil conditions and piling arrangement under which the tests were made:

(*a*) For structures built on timber piles in which lateral movement of not more than $\frac{1}{4}$ in. is allowable, a maximum lateral load of not more than 4 tons per pile should be allowed if the piles are subject to fatigue by frequent repetitions or reversals of load, or 4.5 tons per pile if the load is merely to be sustained;

(*b*) For structures built on timber piles in which a lateral movement of not more than $\frac{1}{2}$ in. is allowable, a maximum lateral load of not more than 6.5 tons per pile should be allowed if the piles are subject to fatigue by frequent repetitions or reversals of load, or 7.0 tons if the load is to be sustained;

(*c*) Walls supported by piles having adequate vertical bearing capacity, when subjected to lateral loads as great as 20 tons per pile, will remain vertical when moved in a horizontal direction;

(*d*) Fixing the heads of the piles is essential in order to determine, accurately, the movement of structures under lateral loads;

(*e*) Frequent repetition of lateral loads results in slightly greater lateral movement than a sustained load of the same magnitude;

(*f*) For lateral unit loads as great as about 6.5 tons per pile on a group of piles arranged in two rows, as under the test monoliths, the total resistance to movement increases in direct proportion to the number of piles. (As the unit lateral load increases above about 6.5 tons per pile, this no longer applies and the resistance per pile becomes less as the number of piles in the group increases); and

(*g*) The use of concrete piles of the type tested under Monolith No. 6, will permit increasing the designed lateral loads given in Conclusions (*a*) and (*b*) by from 1 ton to 2 tons per pile.

ACKNOWLEDGMENTS

The tests herein described were made in connection with the construction by the United States Government of the Twin Locks of Lock and Dam No. 26, which were designed under the supervision of W. H. McAlpine, M. Am. Soc. C. E. The construction work is under the supervision of Captain B. M. Harloe, Corps of Engineers, U. S. Army, District Engineer, St. Louis District, with Captain W. W. Wanamaker, Corps of Engineers, U. S. Army, Military Assistant, in local charge.

The writer wishes to acknowledge the able assistance of Messrs. C. E. Wuerpel, Assistant Engineer, and G. A. Allen, Junior Engineer, in making the field observations and assembling data, and of J. A. Adams, Associate Engineer, in preparing the illustrations accompanying this paper. Acknowledgment is especially made of the helpful suggestions of Captain Wanamaker, who conceived the idea of making the tests. The test monoliths were constructed and the tests were performed by Bruce Gordon, Superintendent for the John Griffiths and Son Company, General Contractor for the construction of the Twin Locks.

(c) The use of concrete piles of the type tested under identical lateral loads will permit increasing the allowable lateral load stated in Conclusions (a) and (b) by from 1 ton to 2 tons per pile.

Acknowledgments

The tests herein described were made in connection with the construction by the United States Government of the Twin Locks of Lock and Dam No. 26 which were designed under the supervision of W. H. McAlpine, M. Am. Soc. C. E. The construction work is under the supervision of Captain R. M. Gooding, Corps of Engineers, U. S. Army, District Engineer at St. Louis District, with Captain W. N. Wannamaker, Corps of Engineers, U. S. Army, Military Assistant, in local charge.

The writer wishes to acknowledge the able assistance of Messrs. C. E. Wuerpel, Assistant Engineer, and E. A. Allen, Junior Engineer, in making the field observations and assembling data, and of R. L. Adams, Associate Engineer, in preparing the illustrations accompanying this paper. Acknowledgment is especially made of the helpful suggestions of Captain Wannamaker, who conceived the idea of making the tests. The test piles were constructed and the tests were performed by Frank Gordon, Superintendent for the John Griffiths and Son Company, General Contractor for the construction of the Twin Locks.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

LATERAL PILE-LOADING TESTS

Discussion

By A. E. Cummings, Assoc. M. Am. Soc. C. E.

A. E. Cummings,[2] Assoc. M. Am. Soc. C. E. (by letter).[2a]—A considerable store of valuable information on the lateral deflection of foundation piles is contained in this paper. The writer is fortunate in having had a number of opportunities to discuss these tests with the author and the method of analysis herein proposed was developed for the particular purpose of attempting to check, theoretically, the experimental data presented.

The assumption is made that the upper end of the pile is embedded deeply enough in the foundation to be fixed against rotation. It is also assumed that the lower part of the pile is fixed and that some unknown length, *L*, of the upper part is bent into a deflection curve as shown in Fig. 17(*a*). The

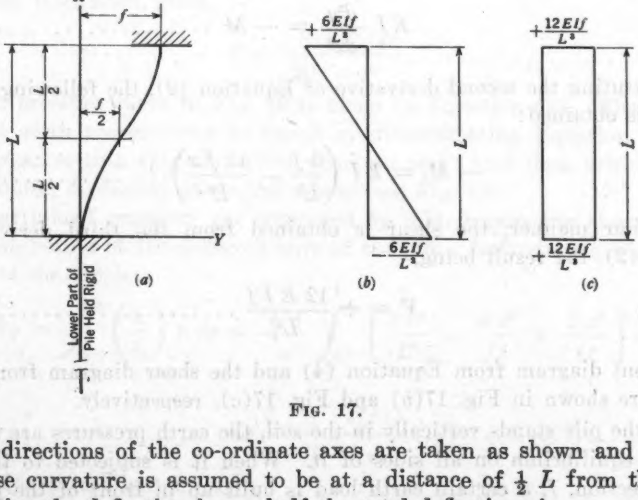

Fig. 17.

positive directions of the co-ordinate axes are taken as shown and the point of reverse curvature is assumed to be at a distance of $\frac{1}{2}$ *L* from the top, at which point the deflection is one-half the total deflection, *f*.

Note.—The paper by Lawrence B. Feagin, Assoc. M. Am. Soc. C. E., is published in November, 1935, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[2] Dist. Mgr., Raymond Concrete Pile Co., Chicago, Ill.

[2a] Received by the Secretary August 1, 1935.

The pile is considered first as a free standing column and the surrounding soil is ignored temporarily. It is assumed that the equation of the central line of the deflected pile can be expressed by the power series[3]:

$$y = a_1 \left(\frac{x}{L}\right)^2 + a_2 \left(\frac{x}{L}\right)^3 + a_3 \left(\frac{x}{L}\right)^4 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

It is then necessary to determine the coefficients of the series in such a way that the boundary conditions of the problem will be satisfied and the resulting equation will represent the elastic curve of the deflected pile. The assumptions as to the shape of the deflection curve furnish the following boundary conditions, from which these coefficients are determined: (1) Since the deflection curve is to have a vertical tangent at the top and at the bottom, the first derivative of Equation (1) is zero at $x = 0$ and at $x = L$; (2) since the point of inflection is to be at the middle of the deformed part of the pile the second derivative of Equation (1) is zero at $x = \frac{L}{2}$; and (3) the deflection is f at $x = L$. These three sets of boundary conditions serve to establish the equation:

$$y = \frac{3 f x^2}{L^2} - \frac{2 f x^3}{L^3} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

which is the equation of the deflection curve of the bent pile.

Equation (2) is then differentiated, successively, three times. By using the differential equation of the elastic curve of a bent beam:

$$E I \frac{d^2y}{dx^2} = - M \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

and substituting the second derivative of Equation (2), the following moment equation is obtained:

$$- M_f = E I \left(\frac{6 f}{L^2} - \frac{12 f x}{L^3}\right) \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

In a similar manner, the shear is obtained from the third derivative of Equation (2), the result being,

$$V = + \frac{12 E I f}{L^3} \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The moment diagram from Equation (4) and the shear diagram from Equation (5) are shown in Fig. 17(*b*) and Fig. 17(*c*), respectively.

When the pile stands vertically in the soil, the earth pressures are approximately in equilibrium on all sides of it. When it is subjected to the horizontal deflection, f, a certain earth load is built up in front of the pile due to the passive resistance of the soil. It will be assumed that this earth load at any point is equal to the deflection times the elastic modulus of the soil. It will also be assumed that the elastic modulus of the sand increases directly

[3] "Drang und Zwang", von Föppl, Vol. 2, p. 312.

with the depth.[4] With the positive directions of the co-ordinate axes chosen as shown in Fig. 18, the equation of the elastic modulus of the soil may be written:

$$E_s = \frac{U\,(L - x)}{\omega} \quad \text{..........(6)}$$

in which U = the weight per unit volume of soil and ω = a dimensionless coefficient determined by the elastic property of the soil.

The unit earth pressure on the front side of the deflected part of the pile is then given by,

$$p = E_s\,y \quad \text{..............(7)}$$

in which y is the deflection curve of Equation (2) and the assumption is made that the shape of this curve is not materially changed by the earth load.

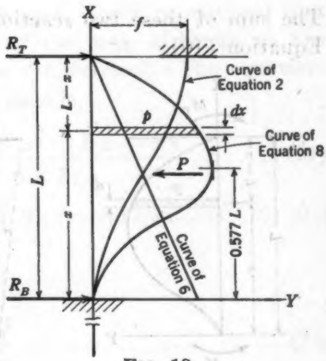

FIG. 18.

This unit pressure is:

$$p = \frac{U\,(L - x)}{\omega}\left[\frac{3\,f\,x^2}{L^2} - \frac{2\,f\,x^3}{L^3}\right] \quad \text{..............(8)}$$

The total earth load is, then,

$$P = \int_0^L p\,dx = \frac{U\,f}{\omega}\int_0^L \left[\frac{3\,x^2}{L} - \frac{5\,x^3}{L^2} + \frac{2\,x^4}{L^3}\right] dx \quad \text{.........(9)}$$

which when integrated, gives,

$$P = \frac{3}{20}\,\frac{U\,f\,L^2}{\omega} \quad \text{......................(10)}$$

The earth pressure curve in Fig. 18 is given by Equation (8). The point of maximum earth pressure can be found by differentiating Equation (8) with respect to x; setting this derivative equal to zero; and then solving for x. The maximum is slightly above the center (see Fig. 18).

The earth-load reactions are computed by taking moments about the top and bottom points of the deflected part of the pile. Referring to Fig. 18, the reaction at the top is:

$$R_T = \int_0^L \left(\frac{x}{L}\right) p\,dx = \frac{U\,f}{\omega}\int_0^L \left[\frac{3\,x^3}{L^2} - \frac{5\,x^4}{L^3} + \frac{2\,x^5}{L^4}\right] dx \quad \text{...(11)}$$

or,

$$R_T = \frac{1}{12}\,\frac{U\,f\,L^2}{\omega} \quad \text{......................(12)}$$

The reaction at the bottom is:

$$R_B = \int_0^L \frac{(L - x)}{L}\,p\,dx = \frac{U\,f}{\omega}\int_0^L \left[\frac{3\,x^2}{L} - \frac{8\,x^3}{L^2} + \frac{7\,x^4}{L^3} - \frac{2\,x^5}{L^4}\right] dx \quad \text{..(13)}$$

[4] "Druckverteilung im Baugrunde", von O. K. Froehlich, p. 89.

or,

$$R_B = \frac{1}{15} \frac{U f L^2}{\omega} \quad \text{......................} (14)$$

The sum of these two reactions is equal to the total earth load as given by Equation (10).

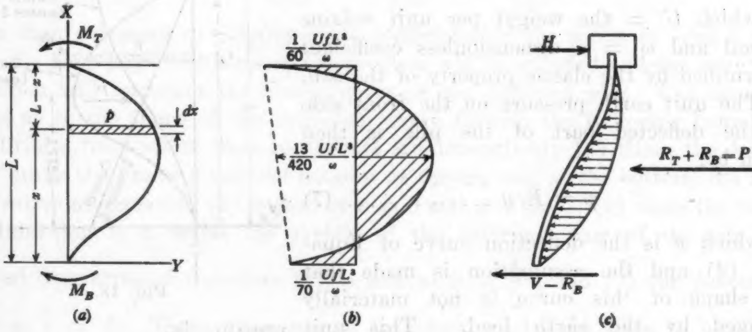

FIG. 19.

To calculate the bending moments and deflections caused by the earth load, use will be made of the method of super-position.[5] Referring to Fig. 19(*a*), the moment due to the couple at the top is:

$$M_T = \int_0^L p \, dx \, \frac{x^2 (L - x)}{L^2} \quad \text{......................} (15)$$

Substituting the value of p from Equation (8) and integrating gives,

$$M_T = \frac{1}{60} \frac{U f L^3}{\omega} \quad \text{......................} (16)$$

The moment due to the couple at the bottom is:

$$M_B = \int_0^L p \, dx \, \frac{x (L - x)^2}{L^2} \quad \text{......................} (17)$$

Substituting the value of p and integrating gives:

$$M_B = \frac{1}{70} \frac{U f L^3}{\omega} \quad \text{......................} (18)$$

The moment produced by the distributed earth load when the pile is considered as a simple beam is,

$$M_S = \int_0^L p \, dx \, \frac{x (L - x)}{L} \quad \text{......................} (19)$$

Substituting for p and integrating,

$$M_S = \frac{13}{420} \frac{U f L^3}{\omega} \quad \text{......................} (20)$$

[5] "Strength of Materials", by S. Timoshenko, Vol. 1, p. 209.

Combining the moment diagram from Equation (20) with the diagram from the two end couples, produces the combined moment diagram shown in Fig. 19(*b*).

In computing the deflections due to the earth load, it will be assumed that the maximum deflection occurs at the center of the span although as has already been shown the earth load is not quite symmetrical. The equation[6] of the deflection curve due to the couples at the ends is,

$$\Delta_c = \frac{M_T\,(x)\,(L^2 - x^2)}{6\,E\,I\,L} + \frac{M_B\,(L - x)\,(2\,L\,x - x^2)}{6\,E\,I\,L} \quad \ldots\ldots\ldots (21)$$

Substituting $x = \frac{L}{2}$ and the values of M_T and M_B from Equations (16) and (18) gives, at $x = \frac{L}{2}$:

$$\Delta_c = \frac{13}{6\,720}\,\frac{U\,f\,L^5}{E\,I\,\omega} \quad \ldots\ldots\ldots (22)$$

The deflection[7] at the center of the span due to the distributed earth load and considering the pile as a simple beam, for $x = \frac{L}{2}$, is:

$$\Delta_S = 2\int_0^{\frac{L}{2}} p\,dx\,\frac{x}{48\,E\,I}\,(3\,L^2 - 4\,x^2) \quad \ldots\ldots\ldots (23)$$

Substituting the value of p from Equation (8) and integrating $\left(\text{for } x = \frac{L}{2}\right)$:

$$\Delta_S = \frac{89}{43\,008}\,\frac{U\,f\,L^5}{E\,I\,\omega} \quad \ldots\ldots\ldots (24)$$

It should be noted that these deflections are in opposite directions and that they are approximately equal since the numerical coefficient of Equation (22) is 0.00193, whereas that of Equation (24) is 0.00207.

One of the quantities measured in the experiments was the jack load. This is an external horizontal force applied near the top of the pile, and it is resisted partly by shear in the pile itself and partly by the earth load built up in front of the pile. Referring to Fig. 19(*c*), the equilibrium condition for the horizontal forces is,

$$H = (V - R_B) + (R_T + R_B) = V + R_T \quad \ldots\ldots\ldots (25)$$

Substituting Equations (5) and (12) into Equation (25) gives:

$$H = \frac{12\,E\,I\,f}{L^3} + \frac{1}{12}\,\frac{U\,f\,L^2}{\omega} \quad \ldots\ldots\ldots (26)$$

Equation (26) indicates that the deflection is a linear function of the external load so that the work done during deflection may be expressed by,

$$W = \frac{H\,f}{2} \quad \ldots\ldots\ldots (27)$$

[6] "Strength of Materials", by S. Timoshenko, Vol. 1, p. 168.

[7] *Loc. cit.*, p. 164.

Substituting Equation (26) into Equation (27) gives:

$$W = \frac{6\,E\,I\,f^2}{L^3} + \frac{1}{24}\,\frac{U\,f^2\,L^2}{\omega} \quad \text{(28)}$$

The assumption is then made that during deflection the pile follows the principle of least action and that the bent length, L, adjusts itself so as to make the total work of deformation a minimum. Differentiating Equation (28) with respect to L:

$$\frac{\partial W}{\partial L} = -\frac{18\,E\,I\,f^2}{L^4} + \frac{1}{12}\,\frac{U\,f^2\,L}{\omega} \quad \text{(29)}$$

The condition for a minimum is,

$$-\frac{18\,E\,I\,f^2}{L^4} + \frac{1}{12}\,\frac{U\,f^2\,L}{\omega} = 0 \quad \text{(30)}$$

Solving for L gives:

$$L = \sqrt[5]{\frac{216\,E\,I\,\omega}{U}} \quad \text{(31)}$$

This is a minimum since the second derivative of Equation (28) with respect to L is positive.

The foregoing theoretical equations will be used to check some of the test results on Monoliths Nos. 2, 3, and 5, of Mr. Feagin's paper. These monoliths contained only wood piles, for which a Young's modulus was determined and for which the moment of inertia can easily be computed. The average diameter at a point 5 ft below the heads of all the piles in these three monoliths is found to be 12.077 in., or 1.0064 ft. The moment of inertia of a circular area of this diameter is 0.0503 ft.^4 The Young's modulus of one of the wood piles was determined as 1 878 260 lb per sq in., or 270 469 440 lb per sq ft. The average weight of the sand on this site is 112.5 lb per cu ft, dry. The percentage of voids varies from 30 to 37 so that in a thoroughly saturated state the sand surrounding these piles weighs about 133 lb per cu ft. This is a very dense sand[8] and the dimensionless coefficient, ω, will be taken as 0.005.

Using these numerical values in Equation (31), the calculated length of the deformed part of the pile is 10.2 ft. With this value of L and with the same values of E, I, U, and ω, a series of calculations was made with Equation (26). For loads of 4, 8, 12, 16, and 20 tons per pile, the theoretical deflections were computed and the results are plotted as a dotted line in Fig. 20. The observed load-deflection curve (which is the average of the data for Monoliths Nos. 2, 3, and 5, taken from Tables 1 and 2) is plotted on the same diagram. It is seen that the computed curve falls somewhat below the observed curve, but, in general, the variations between the two are no greater than those among the observations themselves. The computed curve is on the safe side. In two other ways there are indications of agreement between observation and theory. Mr. Feagin mentions a measurement of the deformed length of one of the piles and reports this as about 10 ft,

[8] "Erdbaumechanik", by Charles Terzaghi, M. Am. Soc. C. E., pp. 11-12, and Table 24, p. 92.

although the measurement was made on a concrete pile. He also mentions the fact that the earth in front of the pile provided the greater proportion of the resistance against deflection, which conclusion is checked by Equation (26).

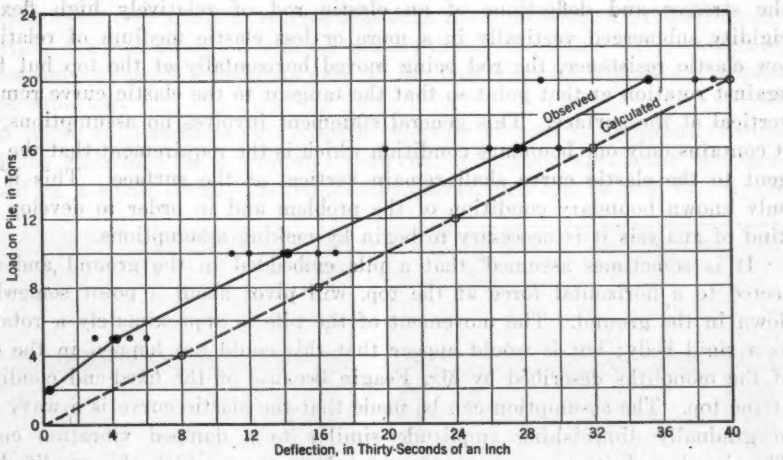

Fig. 20.

Fig. 21(*a*) shows a combined moment diagram at a load of 16 tons per pile, which corresponds to a theoretical deflection of 1 in., the values of

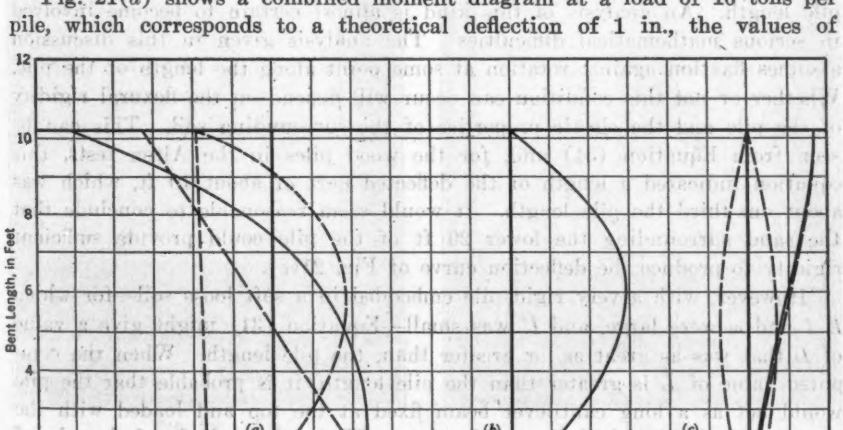

Fig. 21.

L, *E*, *I*, *U*, and ω being the same as before. Fig. 21(*b*) is an earth-load diagram for the same numerical values, and Fig. 21(*c*) is a combined deflection diagram for the same conditions, the solid line representing the final deflection.

In any analysis of this kind it is necessary to make some assumptions. The analysis having been completed and having been applied to experimental data it is usually desirable to reconsider the basic assumptions. Stated in general terms, this problem may be said to involve the determination of the stresses and deflections of an elastic rod of relatively high flexural rigidity submerged vertically in a more or less elastic medium of relatively low elastic resistance, the rod being moved horizontally at the top but fixed against rotation at that point so that the tangent to the elastic curve remains vertical at the surface. This general statement involves no assumptions, but it contains only one boundary condition which is the requirement that the tangent to the elastic curve shall remain vertical at the surface. This is the only known boundary condition of the problem and in order to develop any kind of analysis it is necessary to begin by making assumptions.

It is sometimes assumed[9] that a pile embedded in the ground and subjected to a horizontal force at the top, will pivot about a point somewhere down in the ground. The movement of the pile is approximately a rotation as a rigid body; but it would appear that this could not happen in the case of the monoliths described by Mr. Feagin because of the fixed-end condition at the top. The assumption can be made that the elastic curve is a wavy line of gradually diminishing amplitude similar to a damped vibration curve. This involves further assumptions as to the rate at which the amplitude is damped out and the number of wave lengths that shall be included in the pile length. An analysis of this kind is almost certain to become involved in serious mathematical difficulties. The analysis given in this discussion assumes fixation against rotation at some point along the length of the pile. Whether or not this condition can occur will depend on the flexural rigidity of the pile and the elastic properties of the surrounding soil. This can be seen from Equation (31) and, for the wood piles in the Alton tests, this equation indicated a length of the deflected part of about 10 ft, which was about one-third the pile length. It would seem reasonable to conclude that the sand surrounding the lower 20 ft of the pile could provide sufficient rigidity to produce the deflection curve of Fig. 21(*c*).

However, with a very rigid pile embedded in a soft loose soil—for which $E\,I$ and ω were large, and U was small—Equation (31) might give a value of L that was as great as, or greater than, the pile length. When the computed value of L is greater than the pile length, it is probable that the pile would act as a long cantilever beam fixed at the top and loaded with the earth load developed during deflection. When the calculated length of the deflected part of the pile is of the order of two-thirds or three-fourths of the pile length, it is improbable that the lower end of the pile could remain fixed against rotation. The bottom of the pile would probably move in a direction opposite to that of the deflection at the surface. In either case it would be necessary to revise the fixed-end assumption and to repeat the analysis with some other assumed deflection curve.

It should be noted that Equation (31) provides a means of determining whether or not the fixed-end assumption is justified. It should also be noted

[9] *Civil Engineering*, December, 1934, p. 622.

that this equation does not contain the deflection, but it should not be concluded from this that the bent length is independent of the deflection. Equation (31) is a combination of Equations (5) and (12). Equation (5) is based on the usual theory of elastic structures in which the displacements are required to be very small in comparison with the dimensions of the structure. Equation (12) is based on similar assumptions. Equation (31), therefore, applies only to small deflections even if the deflection itself does not appear in the equation.

In order to determine the relative behavior of rods of different flexural rigidities embedded in a granular material, a simple experiment was performed on a small scale. The results are shown in Figs. 22 and 23. Three

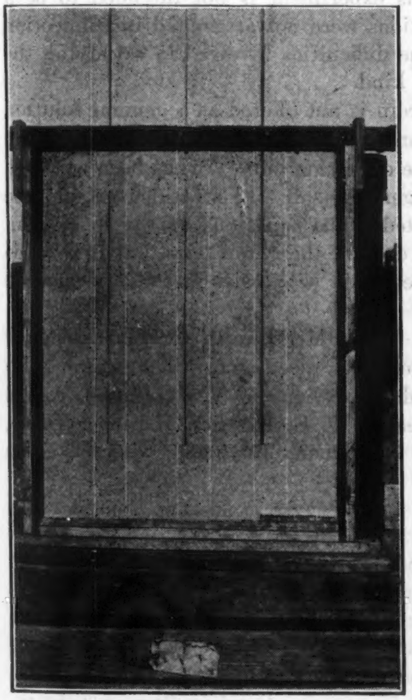

Fig. 22.

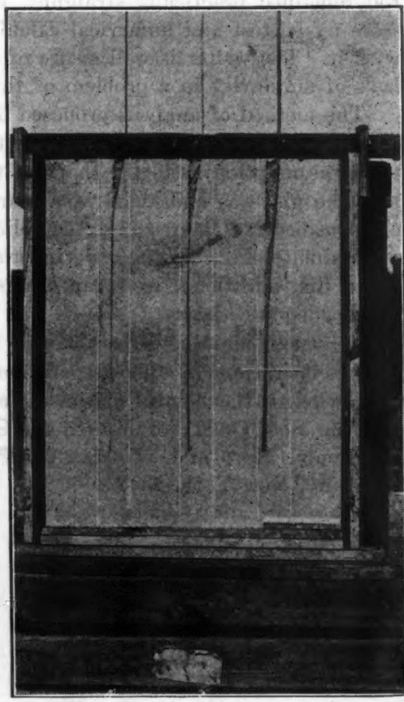

Fig. 23.

steel rods were placed in a vertical position inside the glass front of the box and dry sawdust was then poured in and tamped lightly. The sizes of the rods, from left to right, were $\frac{3}{32}$ in. round, $\frac{1}{8}$ in. square, and $\frac{1}{4}$ in. square. The upper ends of the rods were fixed against rotation by being clamped between two notched steel bars, and vertical lines were marked on the glass for comparison as shown in Fig. 22.

The steel bars holding the upper ends of the rods were then moved slowly about 1 in. to the right with the results shown in Fig. 23. This was an excessive deflection but it was used in order to produce sufficient movement

to photograph. Each rod deflected in a reverse curve and the horizontal lines on the glass mark the bottoms of these curves. The smallest rod had a very short deflection curve, and there was no apparent lateral movement of the lower part of the rod. The middle rod had a longer deflection curve and just below the curve there was a slight movement of the rod toward the left. This movement was very small in comparison with the deflection at the top and it did not become apparent until the top deflection was nearly an inch. The deflection curve of the largest rod amounted to almost three-fourths of the embedded length of the rod. The lower part of the rod was incapable of maintaining its vertical position and the bottom of the rod moved about $\frac{1}{4}$ in. to the left. As nearly as could be determined the lower part of the rod remained practically straight. This experiment is not intended to be a scale model test and numerical calculations were not attempted in connection with it. The writer is well aware of the difficulties involved in satisfying the laws of similarity in a problem of this kind.

The method of analysis proposed herein is not offered as a general solution of the problem of the lateral deflection of a foundation pile. In addition to the assumption as to the shape of the deflection curve, it has been assumed that the pile was embedded in a homogeneous soil. In stratified ground the problem becomes increasingly complicated. The equations appear to give an approximate check on the experimental data of the Alton tests, but it is to be hoped that additional tests can be made which will assist in the development of an adequate theory.

In conclusion, the writer wishes to thank Mr. Feagin for his assistance and co-operation in the preparation of this discussion. The writer also acknowledges the helpful criticisms and suggestions of V. P. Jensen, Assoc. M. Am. Soc. C. E.; of Mr. S. M. Gleaser, U. S. Division Engineer Office, St. Louis, Mo.; and of Mr. W. P. Kinneman, Raymond Concrete Pile Company, New York, N. Y.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

DISCUSSIONS

RELATION BETWEEN RAINFALL AND RUN-OFF FROM SMALL URBAN AREAS

Discussion

BY W. W. HORNER, M. AM. SOC. C. E., AND
F. L. FLYNT, ASSOC. M. AM. SOC. C. E.

W. W. HORNER,[17] M. AM. SOC. C. E., AND F. L. FLYNT,[18] ASSOC. M. AM. SOC. C. E. (by letter).[18a]—The discussions by Messrs Snyder and Sherman develop the possibilities of an alternate study of the rainfall-run-off relation in terms of losses. In Part I of the paper, the writers made a fairly exhaustive effort to determine the relationship as a ratio. They found the ratios, or percentage values, to vary over a wide range and, for a large part of the data, were unable to allocate, satisfactorily, the variation to other hydrologic factors. As was shown in Fig. 18, all these ratios, whether taken between various summation values, or between values for particular periods (as for 5 or 10 min), or between peak rates, resulted in percentile curves of about the same slope. Each of the discussers has emphasized the point that the percentage factor must necessarily be a varying one. This is a condition which is well recognized by hydrologists, of course. Sewer designers have adhered to the use of a run-off-rainfall factor and the formula of the rational method, however, because of its simplicity of application, and for the reason that the dearth of real basic information on run-off from urban areas appeared to make further refinement in method unjustifiable.

The writers are impressed by the sample demonstration offered by Mr. Snyder, as well as by the reasonableness of Mr. Sherman's presentation. It is entirely possible that an alternate analysis of all the information used by the writers, involving the infiltration losses as determined by Mr. Snyder, and the further evaluation of storage as discussed by Mr. Sherman, might result in a set of relatively consistent values having a definite relationship to such basic characteristics as slope, shape of area, and character of surface

NOTE.—The paper by W. W. Horner, M. Am. Soc. C. E., and F. L. Flynt, Assoc. M. Am. Soc. C. E., was published in October, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1935, by Messrs. Franklin F. Snyder, Merrill M. Bernard, and LeRoy K. Sherman.

[17] Cons. Engr., St. Louis, Mo.

[18] Civ. Engr., formerly with Sewer Design Dept., City of St. Louis, St. Louis, Mo.

[18a] Received by the Secretary September 23, 1935.

coverage. Some of the relationships would probably be of the same general order as those suggested by Mr. Bernard in another paper.[19]

The writers agree with Mr. Snyder that a constant infiltration rate could not be used satisfactorily for rains of less than 1 hr in duration. A desirable approach to such a study, accordingly, would be along the lines of determining infiltration capacities for each of the particular areas (which capacities might be expected to vary on a seasonal basis), and also the probable rate of reduction of infiltration within the first hour or two of the storm. It would be necessary, however, to determine the proportionate part of the precipitation going into (temporary) surface storage during the rise of the hydrograph. This also is determinable, possibly, from the same data.

The recent studies by Robert E. Horton, M. Am. Soc. C. E., referred to by Mr. Sherman,[15] using in part the data presented in this paper (privately published and, unfortunately, not available to this discussion) seem also to indicate that satisfactory basic data of this type could be derived from the information originally analyzed by the writers. If such an analysis was found to produce reasonable values of infiltration and storage, some development of Mr. Sherman's Equation (8) might be substituted for the rational formula of sewer design.

Illustrative of how an analysis of basic rainfall and run-off data, such as are supplied by the St. Louis records, might throw some light upon the variation of infiltration rate and temporary storage or detention, the data for

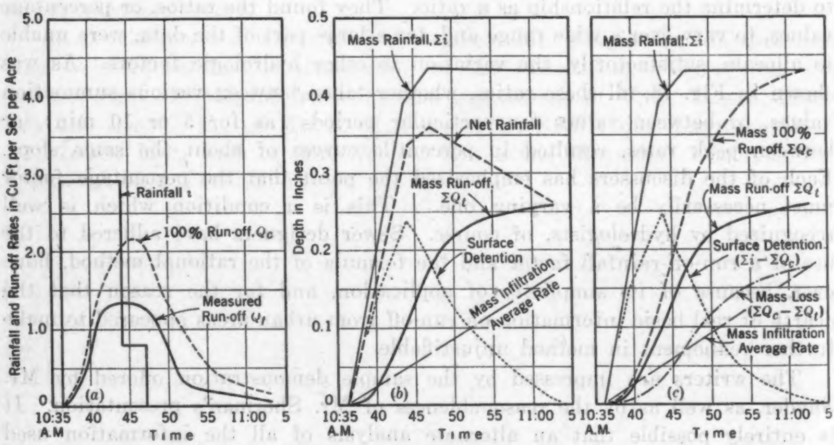

FIG. 31.—STORM OF JULY 23, 1931, AT AREA A.

three storms at Area *A* are presented in Figs. 31, 32, and 33, together with a detailed analysis of the various curves derived from the basic data by various methods. Fig. 31 gives the data for the storm of July 23, 1931, which was chosen for its short duration and unusually uniform rate of precipitation.

[19] "An Approach to Determinate Streams Flow", *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 358.

[15] "Surface Runoff Phenomena", by Robert E. Horton, *Publication 101*. Horton Hydrological Laboratory, February 1, 1935, Edwards Bros. Inc., Ann Arbor, Mich., Publishers.

Fig. 31 (*b*) follows the method suggested by Mr. Horton and referred to by Messrs Snyder and Sherman. The basic data as to rainfall and run-off are represented by the $\Sigma\, i$-curve and the $\Sigma\, Q$-curve, respectively. For simplicity, infiltration is assumed to be at a constant rate and the mass infiltration curve is a straight line through the origin at the beginning of rainfall and with a final ordinate at the end of run-off equal to the difference between total rainfall and total run-off. (The value of "infiltration" thus determined, is in error in that it includes permanent retention and other losses.) The mass net rainfall curve is obtained by subtracting the ordinates to the mass infiltration curve from corresponding ordinates of the mass rainfall, or $\Sigma\, i$-curve. The intercept on a given time ordinate between the net rainfall curve and the mass run-off curve represents the depth of surface detention, including both sheet and channel storage, which, as pointed out by Mr. Sherman, bears a direct relation to the rate of run-off.

On further consideration, the writers find that this rather unsatisfactory approach may be avoided entirely, and the desired item of mass loss and of detention may be determined directly from mass curves based on the unit graph, as originally presented in Fig. 15 of the paper.

Fig. 31 (*c*) illustrates the method of obtaining the depth of surface detention by the application of the unit graph formulas to the same basic data. In this case, the mass 100% run-off, or $\Sigma\, Q_c$-curve, is obtained by applying the unit graph formulas to the rainfall data. The surface detention is then determined by the intercept on the time ordinates between the $\Sigma\, i$-curve and the $\Sigma\, Q_c$-curve.

The surface detention curves as determined by the two methods show a remarkable similarity in essential features despite the radical difference in the methods, which indicates that the unit graph formulas as developed by the writers may be close approximations to the true values. Such being the case, the procedure may be extended to determine the probable variation in the value of the rate of water loss (of which infiltration forms the greater part) during the progress of a storm. The accumulated water loss represented by the mass loss curve, Fig. 31 (*c*), is determined by the intercepts on the time ordinates between the $\Sigma\, Q_c$-curve and the $\Sigma\, Q_t$-curve.

At first glance, the comparatively low rate of water loss during the early minutes of the storm may seem surprising, but further study shows that this is quite reasonable when it is considered that the area in question is composed of both pervious and impervious surfaces and that the first water to reach the inlet (and, in fact, the only water to reach it for some time) is that which has fallen upon the adjacent paved streets, which are highly impervious surfaces. Naturally, then, the loss is low. As the pervious areas (in this case mostly sodded), begin to contribute water to the inlet, the percentage of loss becomes higher and the curve becomes steeper, and, for a considerable period (almost 10 min), the rate of loss is constant. This loss may well represent the average rate for the entire area, pervious and impervious. Soon after the cessation of rainfall, the rate begins to decline, which indicates that the run-off now consists mainly of gutter flow, in which

case, again, the loss would naturally be low due to the impervious surface on which the water is flowing. It seems reasonable to infer that a straight line representing constant infiltration (such as in Fig. 31 (*b*)) would only apply to areas where the entire surface was of the same degree of permeability.

Fig. 32 gives the basic data and derived curves for the storm of September 15, 1914, which may be described as an "average rain" (Fig. 38, introduced subsequently, shows the rainfall pattern).

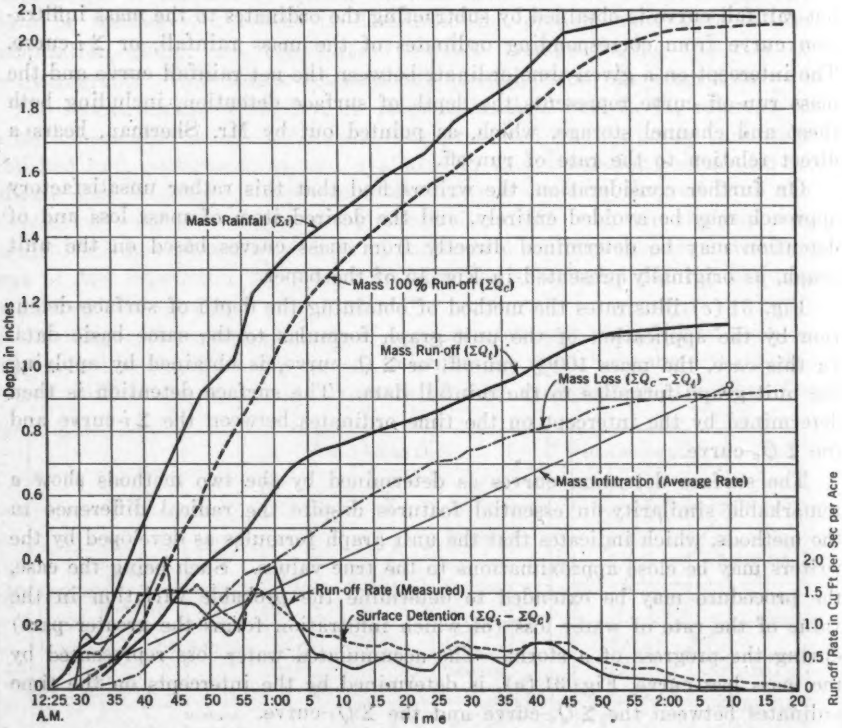

FIG. 32.—STORM OF SEPTEMBER 15, 1914, AT AREA A.

The loss curve in this case shows the same general characteristics as the one shown in Fig. 31 (*c*), the minor differences being explained by the different rainfall pattern. There is again the low initial loss and the high rate of loss during the period of high precipitation, when the pervious areas are contributing their full quota of water. After the period of heavy rainfall the gutter flow, which includes much of the water falling during the heavy rainfall period, tends to reduce the average rate of loss, and, as the rate of rainfall becomes less and less toward the end of the storm, the rate of loss declines due to the predominance of gutter flow.

Fig. 33 shows the same analysis applied to the data for the storm of June 19, 1928, in which there were two periods of heavy rainfall about an hour

apart, with continuous but very light precipitation between them. Here, again, is found the same variation in the rate of loss, which is susceptible to the same explanation given in the foregoing analysis. To the beginning

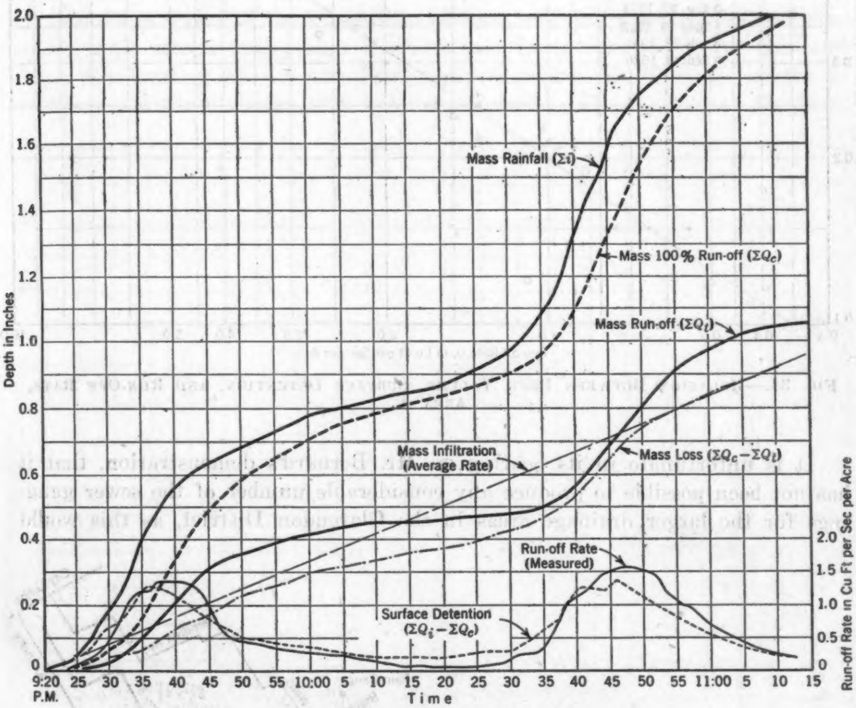

FIG. 33.—STORM OF JUNE 19, 1928, AT AREA *A*.

of the second period of heavy rainfall, the variation follows the same general pattern as in Fig. 32. The sharp rise thereafter is due to the fact that the pervious areas are again contributing a large share of the water appearing as run-off, whereas during the lull between the storms the impervious areas were contributing most of the water due to the fact that the infiltration rate on the pervious area was nearly or quite as great as the rate of precipitation.

In the three storms (Figs. 31, 32, and 33), the close relation between the depth of surface detention and the rate of run-off is obvious. Fig. 34 shows an attempt to arrive at the mathematical correlation between these two factors by plotting the values for depth of surface detention for several peaks, in the aforementioned storms and two others, against corresponding run-off peaks in the same storms. The results are from a small number of storms, and both pervious and impervious areas are represented in varying proportions at different peaks. However, the general trend of the plotted points tends to agree with the equation given by Mr. Sherman: $Q = C\sqrt{S\,d} \times d$, or Q varies as $d^{\frac{3}{2}}$.

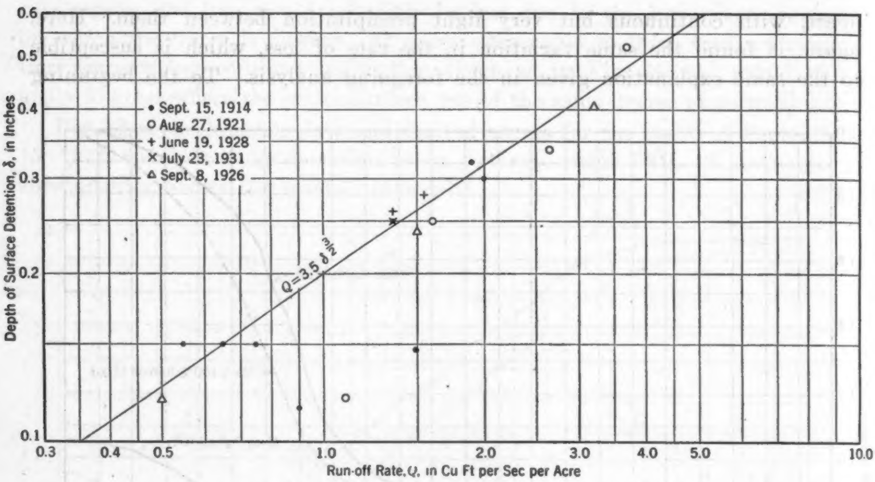

FIG. 34.—RELATION BETWEEN PEAK VALUES, SURFACE DETENTION, AND RUN-OFF RATE, AREA A.

It is unfortunate in its bearing on Mr. Bernard's demonstration, that it has not been possible to produce any considerable number of the sewer gaugings for the larger drainage areas in the Clarendon District, as this would

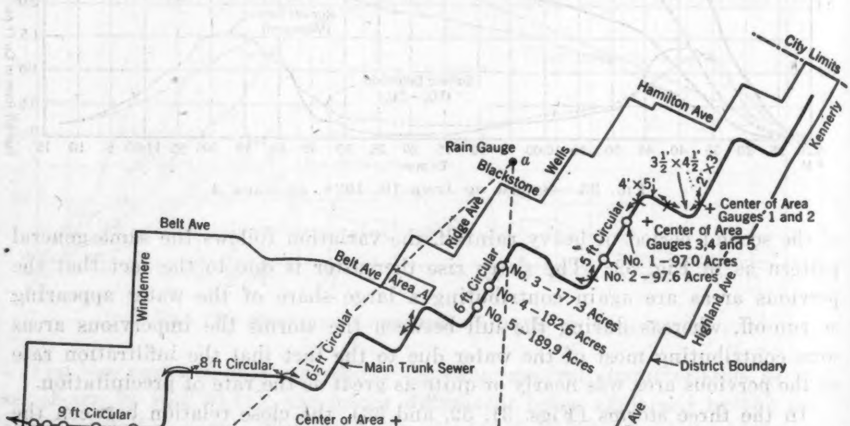

FIG. 35.—CLARENDON SEWER DISTRICT, SHOWING LOCATION OF GAUGES.

have made possible a clearer appreciation of the manner in which run-off rates from individual city blocks are translated into rates applicable to the larger sewers. As an indication of the possibilites, the results for one particular rain are given herewith.

Fig. 35 is a plat of the Clarendon District, showing the location and sizes of the main sewers (the grades of which vary from one-third to two-thirds of 1%), the location of the principal pressure gauges in the main sewers by number, and the drainage area tributary to each. The location of the three

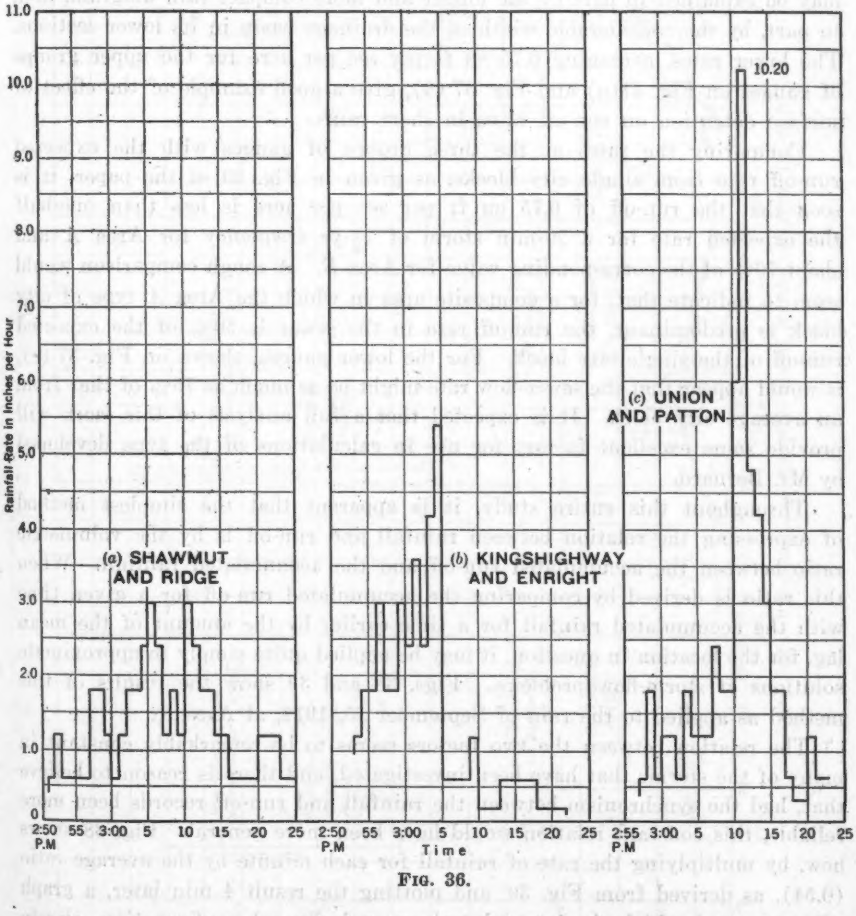

FIG. 36.

tipping-bucket rain-gauges (marked *a*, *b*, and *c*) is also shown on this plat, as well as the location of the two city blocks from which the data for the original paper were collected.

Fig. 36 shows the rain record of July 10, 1920, at three rain-gauges, showing the wide variation in pattern possible even within a 600-acre district. This particular rain also showed a definite tendency to progress up the drain-

age area. Fig. 37 shows the interpolated rain diagram applicable to the centers of the drainage areas above each group of sewer gauges, and, on this same diagram, have been plotted the hydrographs of sewer flow for each gauge in the group. For comparison, the average rainfall rate for the approximate critical time of 20 min is shown in Fig. 37 (*a*) and Fig. 37 (*b*) and the average rate for 25 min is shown in Fig. 37 (*c*). Comparing these rain values with Fig. 23, it would appear that this rain would have a frequency of about $1\frac{1}{2}$ yr.

The higher run-off rates, in cubic feet per second per acre, in Fig. 37 (*c*) may be explained in part by the longer and more compact rain diagram, and, in part, by the considerable width of the drainage basin in its lower sections. The lower rates, averaging 0.75 cu ft per sec per acre for the upper groups of gauges on Fig. 37(*a*) and Fig. 37 (*b*), give a good example of the effect of surface detention on run-off rates in short rains.

Comparing the rates at the three groups of gauges with the expected run-off rate from single city blocks, as given in Fig. 23 of the paper, it is seen that the run-off of 0.75 cu ft per sec per acre is less than one-half the expected rate for a 20-min storm of $1\frac{1}{2}$-yr frequency for Area *A* and about 70% of the corresponding value for Area *B*. A rough comparison would seem to indicate that, for a composite area in which the Area *A* type of city block is predominant, the run-off rate in the sewer is 50% of the expected run-off of the single city block. For the lower gauges, shown on Fig. 37 (*c*), it would appear that the sewer-flow rate might be as much as 75% of that from an average city block. It is expected that a full analysis of this work will provide some excellent factors for use in calculations of the type developed by Mr. Bernard.

Throughout this entire study, it is apparent that the simplest method of expressing the relation between rainfall and run-off is by the volumetric ratio between the accumulated run-off and the accumulated rainfall. When this ratio is derived by comparing the accumulated run-off for a given time with the accumulated rainfall for a time earlier by the amount of the mean lag, for the location in question, it may be applied quite simply to approximate solutions of storm-flow problems. Figs. 38 and 39 show the results of this method as applied to the rain of September 15, 1914, at Area *A*.

The relation between the two factors seems to be remarkably constant in many of the storms that have been investigated, and there is reason to believe that, had the synchronism between the rainfall and run-off records been more reliable, this constant relation would have been more general. Fig. 38 shows how, by multiplying the rate of rainfall for each minute by the average ratio (0.54), as derived from Fig. 39, and plotting the result 4 min later, a graph is constructed which, in its peak values, and general configuration, closely approximates the measured run-off curve. This suggests a simple method for predicting the (approximate) probable run-off from a given rainfall when the mean run-off factor and the lag are known for the location in question. Unfortunately, the run-off factor varies over wide limits from one storm to another, as shown in Fig. 18 of the paper.

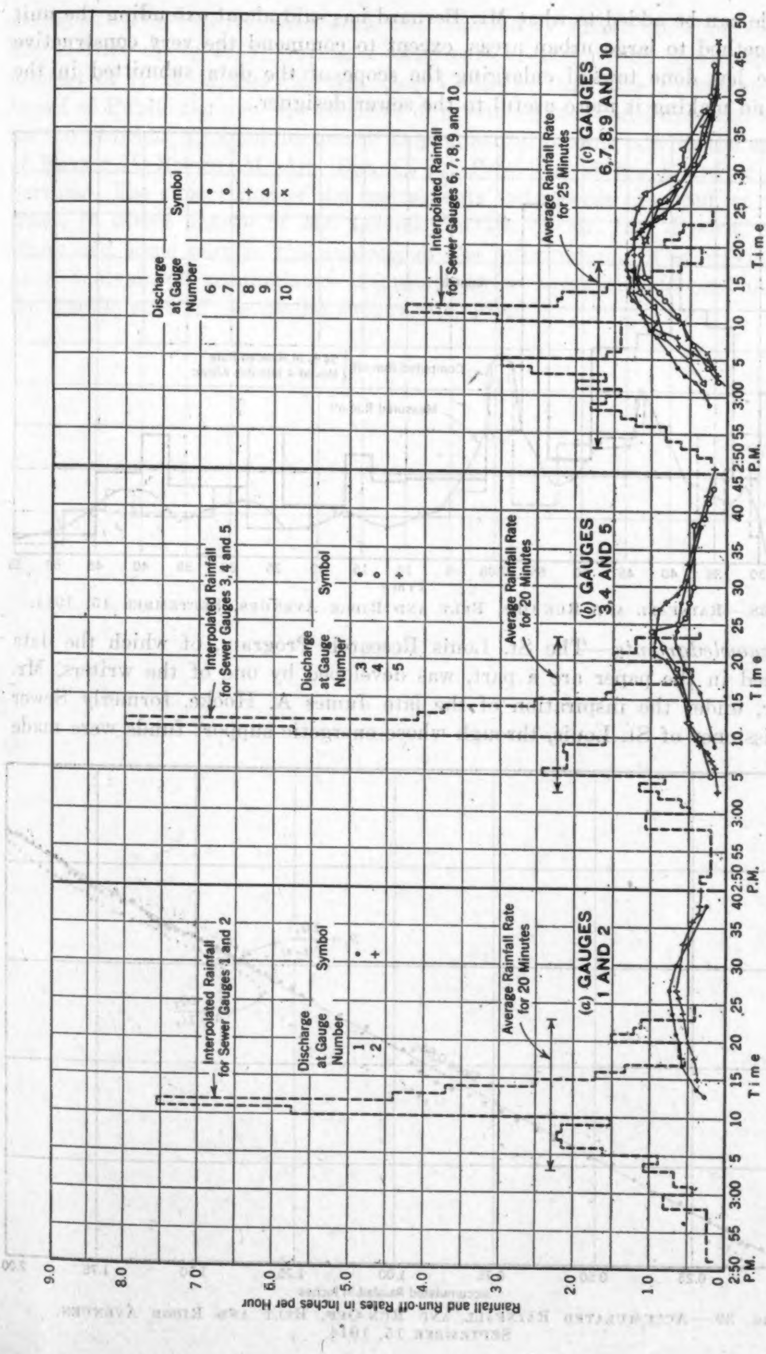

FIG. 37.—RAINFALL AND RUN-OFF DATA, STORM OF JULY 10, 1920.

Little can be added to what Mr. Bernard has said about extending the unit graph method to large urban areas, except to commend the very constructive work he has done toward enlarging the scope of the data submitted in the paper and making it more useful to the sewer designer.

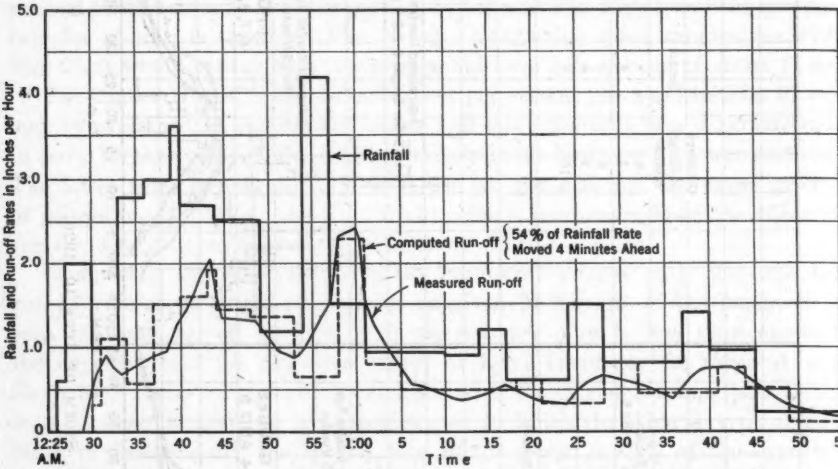

FIG. 38.—RAINFALL AND RUN-OFF, BELT AND RIDGE AVENUES, SEPTEMBER 15, 1914.

Acknowledgments.—The St. Louis Research Program, of which the data presented in the paper are a part, was developed by one of the writers, Mr. Horner, under the inspiration of the late James A. Hooke, formerly Sewer Commissioner of St. Louis, through whose energetic support funds were made

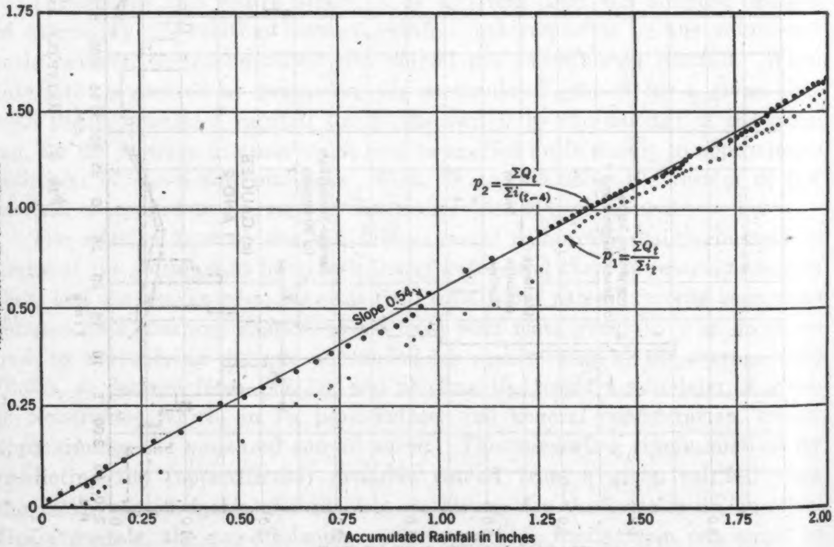

FIG. 39.—ACCUMULATED RAINFALL AND RUN-OFF, BELT AND RIDGE AVENUES, SEPTEMBER 15, 1914.

available. The experimental work was continually under the supervision of Mr. Horner up to 1933; during a large part of this period the sympathetic interest of E. R. Kinsey, M. Am. Soc. C. E., at that time President of the Board of Public Service, was effective in providing continued financial support for the project. The publication of Departmental records is with the approval of Baxter L. Brown, M. Am. Soc. C. E., President of the Board of Public Service. The supervision of the instruments and records has been, at various times, in direct charge of Mr. Leland Chivvis, or Mr. Guy Brown, each of whom had some part in the analysis of the information. The present complete analysis of accumulated information for three city blocks has been the specific work of one of the writers, Mr. Flynt.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

THE SILT PROBLEM

Discussion

By J. C. Stevens, M. Am. Soc. C. E.

J. C. Stevens,[40] M. Am. Soc. C. E. (by letter).[40a]—A great mass of additional data, particularly on the silting of reservoirs, has appeared in the discussion of the writer's paper. In one paper will now be gathered practically all the basic data regarding the silting of the important reservoirs of the world, as well as summarized data as to the quantity of sediment carried by most of its rivers.

Mr. Nickle mildly chides the writer for not including the excellent data gathered in recent years by the Texas Board of Water Engineers, when as a matter of fact every attempt was made to secure them. Special thanks are due to him for Table 9 containing a summary of the sediment transported at seventeen stations on ten Texas streams, mostly over a 6-yr period, and arranged to conform to the writer's Table 6.

It is interesting to note that of all the stations given in Tables 6 and 9, the maximum concentration of suspended sediment is found in Bad River, at Pierre, S. Dak., where for two consecutive years the sediment averaged more than 38 per thousand. None of the Texas streams listed reached this total even for one year, although the 6-yr average for Double Mountain Fork of Brazos River exceeded 24 per thousand, which shows the next highest concentration of silt.

Mr. Nickle also advances additional evidence of the effect of drying silt in reducing its volume. Medina Reservoir silt shrunk to one-half its original volume and doubled its specific weight in five years of intermittent exposure. The specific weight of 70 lb per cu ft of silt in place for Texas reservoirs, when subject to alternate wetting and drying, is not far different from the writer's average value of 65 lb per cu ft, nor that of 62.5 lb per cu ft adopted by Messrs. Fortier and Blaney.[34]

Professor Lane has given a very interesting historical summary of silt investigations, and a clear statement of the several factors affecting sediment

Note.—The paper by J. C. Stevens, M. Am. Soc. C. E., was published in October 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings* as follows: February, 1935, by Harry G. Nickle, Jun. Am. Soc. C. E.; March, 1935, by Messrs. E. W. Lane, and Frank E. Bonner; May, 1935, by Messrs. Morrough P. O'Brien, Harry F. Blaney, W. W. Waggoner, and Philip R. R. Bisschop; September, 1935, by Herman Stabler, M. Am. Soc. C. E.; and October, 1935, by N. C. Grover, M. Am. Soc. C. E.

[40] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[40a] Received by the Secretary October 14, 1935.

[34] "Silt in the Colorado River and Its Relation to Irrigation", by the late Samuel Fortier, M. Am. Soc. C. E., and Harry F. Blaney, Assoc. M. Am. Soc. C. E., *Technical Bulletin* 67, U. S. Dept of Agriculture, 1928, p. 58.

transportation, first to the stream and second by the stream. He also calls attention to the fallacy of the Dupuit theory, namely, that silt is carried in suspension as the result of differences in velocity of adjacent filaments. Turbulence is the real cause, and theory and evidence are now in accord that sediment is held in suspension by the upward velocity components of turbulent flow; that is, by eddy currents.

Mr. Bonner's statement that the capital outlays for reservoirs will be amortized long before they may be rendered useless by silting, and that the solution of the problem may be safely left to posterity, is scarcely a fitting answer to the questions raised. Because of such a major reservoir a civilization comes into being, a virile, complex, pulsating social system. The capital investment in such a civilization may be wiped out and the system destroyed but it can never be amortized in a financial sense.

A policy of *laissez-faire* is entirely unsuited to the exigencies of the situation. The problem should be squarely faced now and research undertaken with a view to its ultimate solution. Mr. Bisschop cites conditions in South Africa that are now threatening the "very existence of productive centers of population." He states that further extensive developments are being limited to those streams least subject to silting. Lake Mentz on Sundays River (1935) has lost 42% of its capacity in 12 yr. Raising the dam has already begun in order to preserve the civilization that is dependent upon it. Amortization of the capital investment is surely not the answer to this problem. Human institutions must continue.

In Table 10, Professor O'Brien has added a wealth of data on silting of reservoirs from the paper by Dr. Fritz Orth, of which the writer was wholly unaware. This table, however, does not give the silt deposited in terms of the inflow, doubtless because the data were not available. Seven reservoirs are cited that have been completely filled with silt, and fifteen that have lost 50% or more of their original capacity. All of them, however, had original capacities of less than 1% of the annual water supply which means that they were little more than diversion dams. The writer doubts whether any such reservoir could actually lose 100% of its capacity. Table 1 does not show any reservoir as having been entirely filled. As the reservoir fills, more and more of the silt is carried through so that there must be left at least a river channel in the new surface of the reservoir, for which condition all silt passes through.

Mr. Blaney calls attention to the fact that a silty stream or canal is quite free from moss and aquatic plants, and that clarifying such a stream will result in a new source of annoyance, that of moss growth. He states that even if it were possible to clarify the Imperial Valley canals some silt should be left in the water to minimize operation troubles from moss growth.

Mr. Waggoner calls attention to the great fall flood of 1861 that affected the entire Pacific Coast from the Willamette Valley, in Oregon, to Los Angeles, Calif. On December 8, 1861, occurred the largest flood known on the Willamette River. That period was marked by a series of wet years from which the present cycle of equally pronounced dryness has emerged. Obviously, silt depositions and sediment transportation respond with marked fluctuations to these climatic and flood variations.

Mr. Stabler asks whether Man's efforts should be directed toward prevention or cure. It seems quite evident that to show any measure of success the labors of Man must be confined to preventative measures. Of course, he can desilt streams and canals at considerable expense and domestic supplies are necessarily desilted, but these are of secondary importance in the great silt problem. To preserve intact the large agricultural areas he must prevent soil loss. He must learn to prepare the land and cultivate it in such a way as to maintain the soil in sites. In the European and Asiatic countries it has been the practice, for centuries, to terrace sloping lands. America has not as yet adopted this practice.

In non-agricultural areas, such as the Western grazing lands, it is doubtful whether extensive structures for soil maintenance are practicable or even desirable except on certain areas that contribute large quantities of silt to reservoired streams.

It is not sufficient merely to construct soil-holding structures. Provision must be made for their continued maintenance; otherwise, the structures deteriorate, and the entire volume of silt accumulated behind them may be loosed by a single storm with much more serious consequences than if it had been allowed to pass in smaller yearly quantities.

Once silt has found its way into a reservoir—except those for domestic supplies—no practicable method has yet been devised of removing it. All Man can hope to do is to reduce the quantity of silt flowing into the reservoir by such means of holding the soil on the area as he may devise. Any structures such as silt dams, bank protection, etc., built for this purpose, should be of permanent type and ample provision should be made for intelligent annual maintenance.

Considerable can be done in stimulating a protective vegetable growth in favorable areas. In unfavorable areas in which the West abounds such efforts are certain to prove futile. Grazing control will have a favorable effect if properly administered although, by and large, the results are likely to prove disappointing. The writer agrees with Mr. Stabler that if the Western grazing lands are managed primarily in the interests of permanency of forage crops for stock production, little more can be hoped for in the way of soil conservation from grazing control.

The data on the Colorado River drainage are most instructive. It would seem that the plateau region deserves extensive study. About 65 000 acres contribute 75% of the silt and only 10% of the water supply; yet all that silt is moved by the 10 per cent. If it were practicable to impound that 10% on the area and use it for irrigation instead of allowing it to carry silt into the Colorado, such an irrigated section would pay handsome dividends even if it never marketed a crop. There would be some justification for subsidizing such a project.

Mr. Grover illuminates, with some new data, the effect of desilting the Colorado River at Boulder Reservoir. In 10 miles of canyon the clear waters issuing from the control gates picked up, and carried in suspension, approximately 4 000 000 tons in 5 months. In 115 miles farther this quantity was trebled, and yet during the same period 100 000 000 tons were left in the reservoir.

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DISCUSSIONS

ELASTIC PROPERTIES OF RIVETED CONNECTIONS

Discussion

BY R. L. MOORE, ESQ.

R. L. MOORE,[21] ESQ., (by letter).[21a]—In view of the emphasis that has been placed on methods of analyzing continuous frames, it seems that the question of joint rigidity in riveted structures has received too little attention. The author of this paper has made a valuable contribution to the knowledge in this field.

From the standpoint of design, the evaluation of the rigidity of the joints in terms of total fixity has certain advantages over the procedure outlined by the author, particularly where the method of moment distribution is used. It may be shown that the distribution factors for any joint are proportional to the product of the $\frac{I}{L}$-values and the degrees of fixity for the members at that joint. It also follows that the carry-over factors at one end of a member are equivalent to one-half the degree of fixity at the other end. After computing fixed-end moments and making adjustments for the estimated degrees of fixity at the joints, the procedure becomes one of simple moment distribution. The author's illustrative problem, shown in Fig. 33, may be solved readily in this manner. On the basis of the results presented in this discussion, however, the writer would alter somewhat the values for the fixed-end moment used. The author has assumed M_{BA} and M_{BC} in Fig. 33, to be the equivalent of about 50% and 70% fixed, respectively. Although the latter percentage is in accord with the writer's tests, the fixity for the clip-angle connection on the 18-in. I-beam, on the same basis, would not be taken at more than about 20 per cent.

NOTE.—The paper by J. Charles Rathbun, M. Am. Soc. C. E., was published in January, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1935, by Ralph E. Goodwin, Assoc. M. Am. Soc. C. E.; May, 1935, by Messrs. Harold C. Rowan, Walter Scholtz, J. F. Baker, L. E. Grinter, and C. R. Young and K. B. Jackson; and August, 1935, by E. Mirabelli, M. Am. Soc. C. E.

[21] Research Engr., Aluminum Research Laboratories, Aluminum Co. of America, New Kensington, Pa.

[21a] Received by the Secretary September 21, 1935.

In the tests upon which these conclusions are based, two-span continuous beams, having a joint or splice at the center support, were used. By comparing the behavior of such specimens with that of an unspliced beam, having 100% continuity, a direct measure of the efficiency of the joints in transmitting bending moments was obtained.

Figs. 39 and 40 show the eight specimens on which tests were made. Specimens 1 and 2 were representative of cases of 100% and 0% continuity,

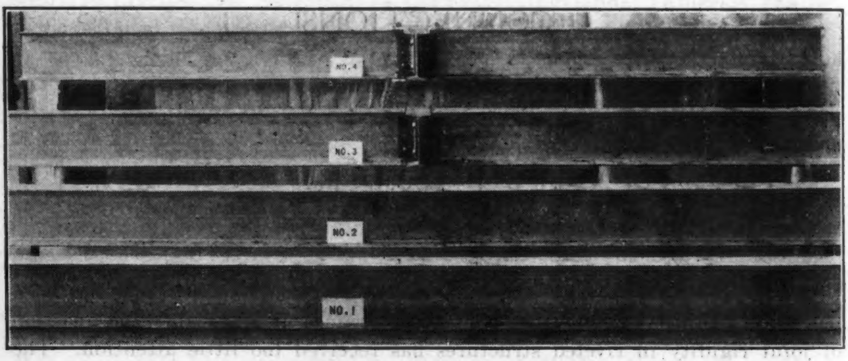

FIG. 39.—SPECIMENS 1, 2, 3, AND 4.

respectively, whereas Specimens 3 to 8 included several common types of beam connections. All specimens were 10 ft 6 in. in over-all length and were

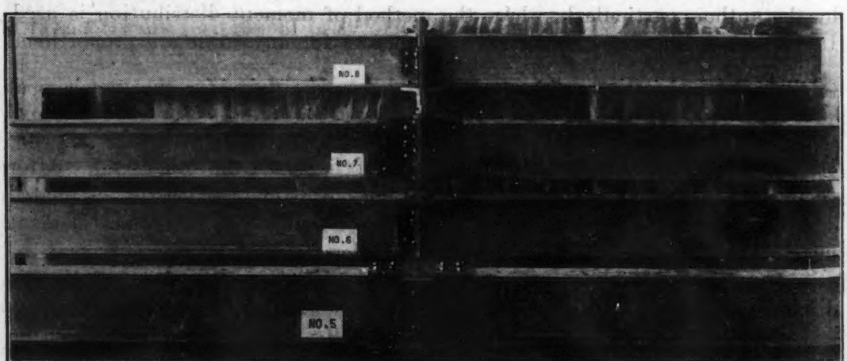

FIG. 40.—SPECIMENS 5, 6, 7, AND 8.

made of 8-in. aluminum alloy I-beams and aluminum alloy angles and plates. The section elements of the I-beams were as follows:

Web thickness, in inches	0.28
Weight, in pounds per foot	5.90
Area, in square inches	4.88
Moment of inertia, I, in inches^4	5.06
Section modulus, S, in inches^3	12.6

TABLE 6.—TENSILE PROPERTIES OF 8-INCH ALUMINUM ALLOY* I-BEAM

Specimen	Yield strength (set = 0.2%), in pounds per square inch	Tensile strength, in pounds per square inch	Percentage elongation, in 2 in.
Web (longitudinal)	43 200	62 900	19.0
Web (transverse)	44 100	65 800	17.0
Flange (longitudinal)	45 400	65 000	20.0

* Duralumin.

The tensile properties are listed in Table 6. The joints were fabricated by means of hot-driven steel rivets $\frac{5}{8}$ in. in diameter. The make-up of each may be described briefly, as follows:

Specimen	Type of Joint
3	Four clip angles, 3 by 3 in. by $\frac{3}{8}$ in. by 6 in. long.
4	Four clip angles, 3 by 3 in. by $\frac{3}{8}$ in. by 6 in. long. Two splice-plates, $\frac{7}{16}$ in. by 4 in. by 8 in.
5	Four clip angles, 3 by 3 in. by $\frac{3}{8}$ in. by 6 in. long. Two splice-plates, $\frac{7}{16}$ in. by 4 in. by 1 ft long.
6	Four clip angles, 3 by 3 in. by $\frac{1}{2}$ in. by 6 in. long. One connection of the joint riveted; the other pin-connected (diameter of pin, $1\frac{1}{2}$ in.).
7	Four clip angles, 6 by 3 in. by $\frac{1}{2}$ in. by 10 in. long.
8	Four clip angles, 3 by 3 in. by $\frac{3}{8}$ in. by 4 in. long. Four seat angles, 3 by 3 in. by $\frac{1}{2}$ in. by 6 in. long.

Fig. 41 shows the nature of the two-span beam tests. The reactions on the auxiliary loading beam of the testing machine used (capacity, 300 000 lb)

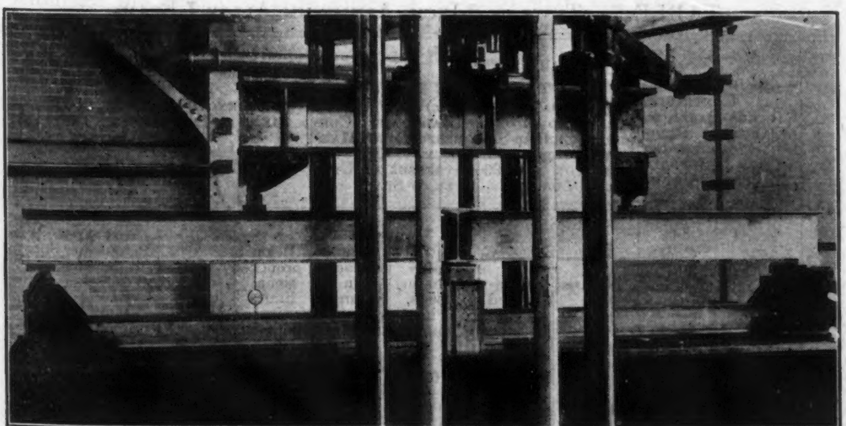

FIG. 41.—TWO-SPAN BEAM TEST OF SPECIMEN 4.

were 5 ft, center to center, whereas the loads on the specimen were applied at the center of each span. Deflections were measured by a dial-gauge graduated to 0.001 in., with respect to a reference beam under the specimen, as shown in Fig. 41. Stresses were determined by a 2-in. strain-gauge, readings

being taken on the top and bottom flanges at a sufficient number of sections to determine the distribution of bending moments.

As a preliminary step in the investigation, deflections and permanent sets were determined for each specimen for loads up to 30 000 lb. The beams were then turned over and the procedure was repeated. The behavior was similar in both cases, and permanent sets were on the order of only 0.002 to 0.004 in.

Fig. 42 shows a set of bending moment curves constructed from the measured stresses for a 50 000-lb load. The ratios of the moments developed

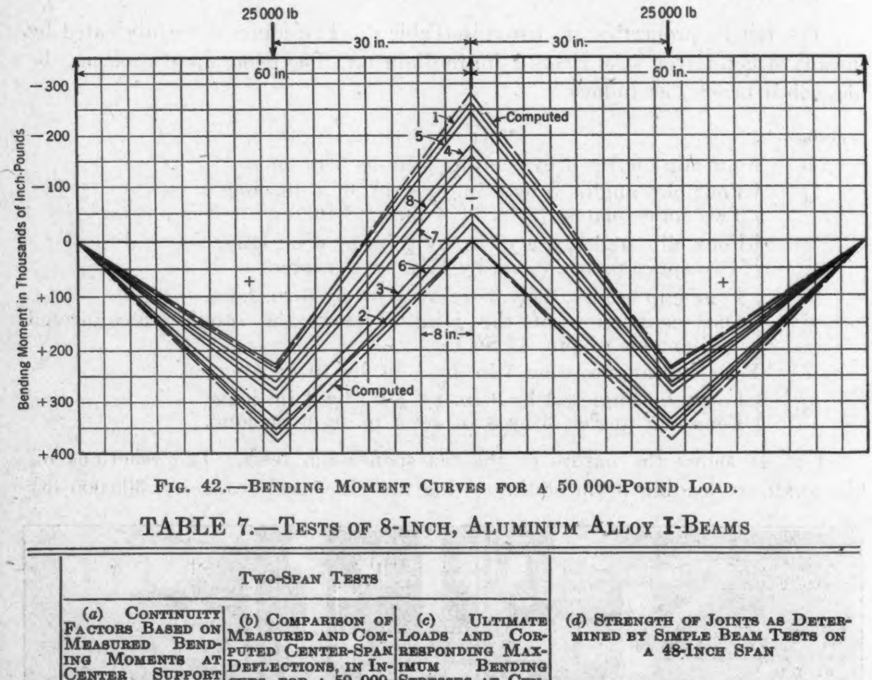

FIG. 42.—BENDING MOMENT CURVES FOR A 50 000-POUND LOAD.

TABLE 7.—TESTS OF 8-INCH, ALUMINUM ALLOY I-BEAMS

Specimen	Two-Span Tests						(d) Strength of Joints as Determined by Simple Beam Tests on a 48-Inch Span			
	(a) Continuity Factors Based on Measured Bending Moments at Center Support for a 50 000-Pound Load		(b) Comparison of Measured and Computed Center-Span Deflections, in Inches, for a 50 000-Pound Load		(c) Ultimate Loads and Corresponding Maximum Bending Stresses at Center of Spans				Maximum Computed Stresses,† in Pounds per Square Inch	
	Bending moment, in inch-pounds	Continuity factors (percentage)	Measured	Computed	Ultimate loads, in pounds	Maximum stresses,* in pounds per square inch	Beam proportional limit, in pounds	Ultimate load, in pounds	At beam proportional limit	At failure
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	258 000	100	0.140	0.145	80 100	29 400	40 000	56 800	37 900	53 800
2	0	0	0.240	0.260	55 500	31 000	......	......	......	......
3	50 000	19	0.231	0.216	62 300	33 100	4 000	8 800‡	63 000	139 000
4	185 000	72	0.170	0.149	69 000	28 600	20 000	37 500	42 000	72 000
5	245 000	95	0.154	0.146	75 500	29 300	30 000	58 800	33 000	67 000
6	30 000	12	0.238	0.234	53 900	30 300	......	......	......	......
7	145 000	56	0.181	0.165	66 000	30 000	15 000	21 800	47 000	68 400
8	160 000	62	0.180	0.157	72 500	30 800	15 000	34 400	36 700	84 000

* Estimated on the basis of average measured stresses for a 50 000-lb load.
† Stresses computed at point where ultimate failure occurred.
‡ Load at which test was stopped; beam continued to yield without fracture.

at the joints over the center supports to those observed in Specimen 1 gave a direct measure of the continuity factors or joint efficiency for each type of construction. From the results given in Table 7 (*a*) it will be noted that none of the joints was the equivalent of an unspliced beam. The highest degree of fixity — 95% — was found for Specimen 5, which had long splice-plates in addition to web clip angles. Continuity factors for the other spliced specimens ranged from 72 to 12%, the latter being found for Specimen 6 which had one span pin-connected near the joint. The standard clip-angle connection in Specimen 3 developed about 19% fixity.

Fig. 43 shows the influence of joint fixity upon the deflections of the different specimens. The relative positions of the curves will be recognized

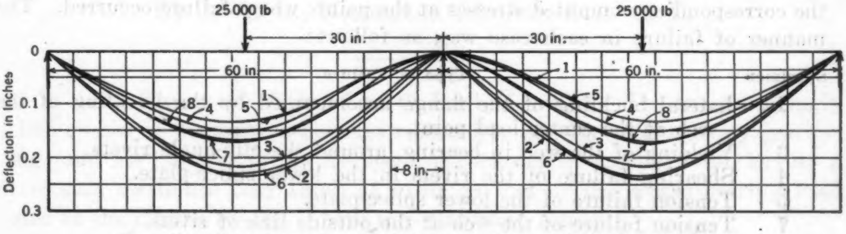

FIG. 43.—COMPARISON OF DEFLECTIONS FOR A 50 000-POUND LOAD.

as about the same as those shown in Fig. 42 for the bending moments. The average measured center-span deflection of Specimen 1, being 100% continuous, was only 58% of that observed for the two simple spans of Specimen 2, whereas all the deflections of the other specimens were between these limits. Although the deflection measurements obtained were not sensitive enough to the degree of fixity at the center support to be used in determining continuity factors, as was done in the case of the bending moments, the agreement between the measured and the computed values given in Table 7 (*b*) was reasonably good.

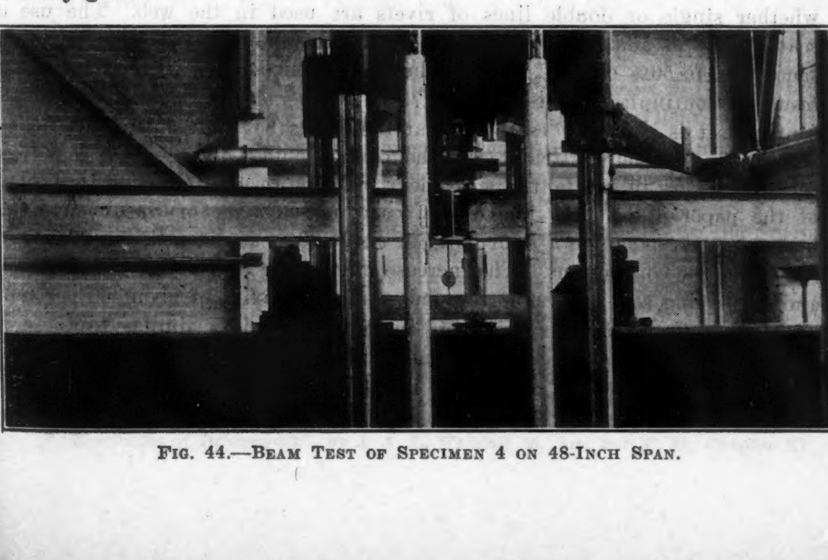

FIG. 44.—BEAM TEST OF SPECIMEN 4 ON 48-INCH SPAN.

In comparing bending moments and deflections for a single 50 000-lb load, attention should be called to the fact that both were proportional to the loads applied up to approximately the point of failure. The latter occurred in every case by a lateral buckling of the compression flanges of the beams without any apparent injury to the joints. Table 7 (*c*) gives the ultimate loads carried and the corresponding maximum computed flange stresses at the centers of the spans.

Since none of the specimens was damaged in the two-span tests all but Specimens 2 and 6, which did not have rigid joints, were subjected to a simple beam test on a 48-in. span as shown in Fig. 44. The purpose of this test was to determine more nearly the ultimate bending resistance of the joints. Table 7 (*d*) gives a summary of the maximum loads carried and the corresponding computed stresses at the points where failure occurred. The manner of failure in each case was, as follows:

Specimen	Type of Failure
1	Lateral buckling of the flange accompanied by the buckling of the web at the center load point.
3	Yielding of the web in bearing, around the clip-angle rivets.
4	Shearing failure of the rivets in the lower splice-plate.
5	Tension failure of the lower splice-plate.
7	Tension failure of the web at the outside line of rivets.
8	Shearing failure of the lower seat-angle rivets.

It is significant that with one exception (Specimen 7) the moments resisted in the simple span tests were from about 70 to 90% higher than those developed in the two-span tests. Furthermore, it appears that this bending resistance may be safely estimated on the basis of the ultimate strength of the material at the point subjected to the maximum stress.

Although it is appreciated that these tests have been limited to comparatively few specimens, the results obtained do indicate some useful limits for design purposes. For standard clip-angle connections it appears that a degree of fixity of from not more than 20 to 40% may be expected, depending upon whether single or double lines of rivets are used in the web. The use of seat angles in addition to clip angles on the web may increase continuity factors up to 50% or 60%, whereas the use of splice-plates on the flanges may result in continuity factors of from 75 to 100 per cent. In using these percentages, it is obviously essential that the joints be made sufficiently strong to carry the moments attributed to them. It should be pointed out that these percentages are in substantial agreement with the results indicated in Table 3 of the paper. The 15% increase in carrying capacity of Specimen 4, for instance, corresponds to a joint fixity of about 20%, as given herein for standard clip-angle connections. The increase in carrying capacity of Specimens 11 and 12, having seat angles, corresponds to a joint fixity of about 60 per cent. Although none of the writer's tests included specimens similar to those in Series *C*, the joint efficiencies obtained for this type of construction ranged from about 65 to 100 per cent.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

WEIGHTS OF METAL IN STEEL TRUSSES

Discussion

BY J. A. L. WADDELL, M. AM. SOC. C. E.

J. A. L. WADDELL,[29] M. AM. SOC. C. E. (by letter).[29a]—The two exceedingly close checks of the writer's findings reported by Mr. Shryock are very gratifying; and his allusion to the European chrome-copper rustless steel, having a corrosion resistance four times as great as that of the copper-bearing steel used in the United States, is both important and timely.

In respect to Mr. Reichmann's suggestion of preparing special curves for continuous trusses, the writer would reply that the way to utilize the curves of the paper for any continuous-span layout is to compute the truss weights as if the spans were non-continuous, then correct the results by means of data published elsewhere.[30]

Replying to Professor Abbett, the curves in Figs. 1 and 2 are intended mainly for rapid preliminary estimates, and are sufficiently accurate in ordinary cases for contractors to use in tendering; but, when there is close competition, it would be advisable to make special computations of truss weights, mainly because the idiosyncrasies of the designer affect the totals.

Mr. Diehl states that "a valuable addition to the paper would be a curve of relative cost of bridges with the conventional types of solid floor as contrasted with open, non-skid flooring." The writer has made a lengthy, complicated, and elaborate investigation of this subject which has not yet (1935) been published. It contains the information mentioned by Mr. Diehl.

Mr. Foight offers a method of comparing truss weights which may prove to be a bit misleading to some readers, causing them to think that the curves are greatly in error. The "percentages" treated in the paper are those of truss weights compared with total vertical loads; hence, the variation in the Springfield Bridge is 2.1% — not 15 per cent. That variation is entirely due to an

NOTE.—The paper by J. A. L. Waddell, M. Am. Soc. C. E., was published in February, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1935, by Messrs. Arthur M. Shaw, Joseph G. Shryock, Albert F. Reichmann, Robert W. Abbett, George C. Diehl, F. G. Jonah, Clarence B. Foight, A. H. Fuller, William E. Wilbur, W. N. Downey, J. R. Grant, Theron M. Ripley, and T. Kennard Thomson; and October, 1935, by Messrs. H. H. Allen and John Venable Hanna.

[29] Cons. Engr. (Waddell & Hardesty), New York, N. Y.

[29a] Received by the Secretary September 25, 1935.

[30] "Economics of Bridgework", by J. A. L. Waddell, M. Am. Soc. C. E., Chapter XI.

uneconomic truss depth. As modern highway bridges are generally designed with economic panel lengths and economic truss depths, no "large errors" can "result from variations in these factors."

Professor Fuller's four cautions to "the young designer and the student who is studying bridge design" are directly to the point, and should be given thorough consideration by all tyros in such work. The writer has never had much luck in determining weights of metal in bridges by formulas, although for half a century he has been using quite freely various "formulas of reduction" of his own in passing from known weights of metal for certain conditions to the corresponding weights for differing conditions.

Replying to Mr. Wilbur, the writer deemed it more logical to make his percentage ratios apply to the grand total vertical load than to that load, less the truss weight; but Mr. Wilbur's supplementary diagrams, based on his assumption of omitting the truss weight itself, are most acceptable. He is correct in stating that it would have been better in Fig. 1 (*c*) and Fig. 1 (*d*) to add to each a curve for a total load of 5 000 lb per lin ft – in truth, the writer has since discovered this fact and has plotted such curves to aid in his own subsequent work. This is a case in which "one's hindsight is better than his foresight."

TABLE 4.—APPROXIMATE AVERAGE PERCENTAGES OF SILICON STEEL MEMBERS IN TRUSSES OF COMBINED SILICON STEEL AND CARBON STEEL.

Roadway width (clear span), in feet	(*a*) PERCENTAGES IN SIMPLE TRUSS BRIDGES, FOR THE FOLLOWING SPAN LENGTHS, IN FEET:					(*b*) PERCENTAGES IN TYPE A CANTILEVER BRIDGES FOR THE FOLLOWING MAIN SPAN LENGTHS, IN FEET:					
	200	300	400	500	600	500	600	700	800	1 000	1 200
20.......	20	70	75	80	82	55	65	70	75	80	85
45.......	35	80	85	90	92	70	75	80	85	90	95

In accordance with Mr. Wilbur's suggestion that the writer include the percentages of silicon steel in structures of combined silicon steel and carbon steel, an attempt is made to furnish the desired information for the trusses in a satisfactory manner. This is especially difficult because of the diverse individual preferences of bridge designers. On that account, tables are preferable to curves; hence, the information is presented in Table 4 as percentages of weight of silicon-steel members compared with that of the truss as a whole. The actual weight of silicon-steel material, however, is from 80 to 90% of the amounts given, since some of the details on silicon-steel members, such as lacing, tie-plates, and rivets, are of carbon steel. Some of the percentages in Table 4 may vary, either up or down in extreme cases by as much as 10% of their values, due to varying conditions and personal peculiarities of the computer, but generally that variation will be less than 5 per cent.

Mr. Wilbur is correct in stating that the loads per foot given in the diagrams are the totals for two trusses, rather than for one truss.

The writer's method of determining the percentage ratios for combined railway-and-highway bridges would be as follows: Let P = the percentage ratio given by the railway-bridge diagram; p = the percentage ratio for the

highway-bridge diagram; $W =$ the equivalent uniform live load, plus impact, per linear foot, for the railway portion of the bridge; $w =$ the equivalent uniform live load, plus impact, for the highway portion of the bridge; and, $p' =$ the desired percentage ratio for the combined bridge. Then,

$$p' = \frac{W\,P + w\,p}{W + w} \qquad (8)$$

Mr. Grant's formula would be:

$$p' = P - \frac{1}{3}(P - p) \qquad (9)$$

If the railway is double-tracked (Class 60); if the clear width of the highway deck is 30 ft with Class A loading; if the structure is of carbon steel; and if the span length is taken as 400 ft,[31] $W = 15\,912$ lb per lin ft; $w = 2\,475$ lb per lin ft, and the curves given in the paper make $P = 25.5$ and $p = 27$. Hence, by Equation (8),

$$p' = \frac{15\,912 \times 25.5 + 2\,475 \times 27}{15\,921 + 2\,475} = 25.7\%$$

and by Equation (9),

$$p' = 25.5 - \frac{1}{3}(25.5 - 27) = 26\%$$

The difference between Mr. Grant's percentage ratio and that of the writer is 0.3.

A similar computation for a single-track railway, a 20-ft clear roadway, and the same span length, gives 27.7% by Equation (9) and 27.3% by Equation (8), a difference of 0.4. Either difference is so small that it would scarcely show on the diagram.

Mr. Ripley's discussion reminds the writer of the days in the early Eighties when he used to consort with the "highwaymen" and compete for highway bridges. Fortunately for him, a call to Japan in 1882 took him out of that objectionable business; and upon his return to the United States four years later, he "declared war" upon all its disreputable practices and initiated what later proved to be a revolution in highway-bridge designing, letting, and construction.

The writer cannot describe in greater detail, "the loadings and other factors that affect Figs. 1 to 5", because the live loads and impacts were presented in another paper,[32] which is readily available. The percentage ratios are given in the present paper; and the manner of using them, both directly and with modifications, is fully explained. The Class A loading used is based on an 18-ton truck, and the Class B loading on a 12-ton truck.

The writer is much indebted to Mr. Allen for his trouble in computing the "percentage ratios" of the numerous bridges he has designed and built, as a check on curves of the paper. The close agreement found is a source of deep

[31] "Bridge Engineering", by J. A. L. Waddell, M. Am. Soc. C. E., pp. 106, 129, 117, and 131.

[32] "Economic Proportions and Weights of Modern Highway Cantilever Bridges", by J. A. L. Waddell, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 888.

satisfaction to the writer. Of course, as Mr. Allen points out, one cannot expect perfect coincidence between the findings of any two engineers, because of the difference in their personal equations. His "percentage ratios" show a marked uniformity, indicating on the part of his computers a close adherence to specifications and the application of the principles of true economy in designing.

A further analysis of Table 2 of the paper, omitting from consideration the abnormal cases of the Springfield Bridge and the Portsmouth lift-span, indicates an average plus variation of 0.19%, an average minus variation of 0.42%, and an algebraic average of all variations equal to minus 0.25 — only one-fourth of 1 per cent. As this analysis is based upon seventeen cases, and in view of the fact that all the fourteen engineers who discussed the paper endorse the curves in general, it may be concluded that the findings of the paper are reliable; nevertheless, it would well pay any engineer who designs many bridges to plot, for the use of his office, new percentage-ratio curves based upon the specifications he utilizes and the general characteristics of his computers.

The writer's experience recently has led him to construct a diagram that should prove useful, especially to beginners, in utilizing Figs. 1 and 2. The

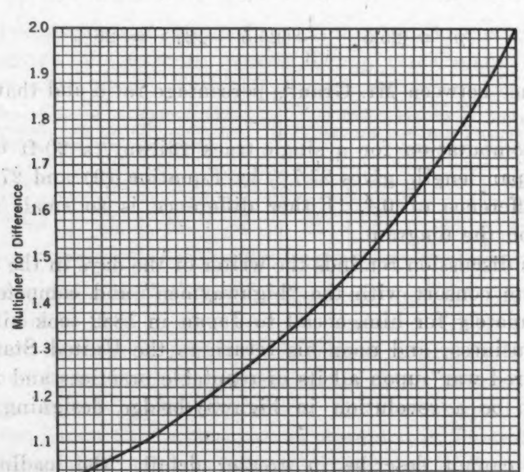

FIG. 15.—CORRECTIONS FOR ASSUMED TRUSS WEIGHTS.

operation in so doing involves an assumption of a truss weight, checking it, assuming another and checking that, etc., until there is an exact agreement between the assumption and the check. The final weight can be found correctly from the first check weight by computing the difference between it and the assumption, and multiplying the result by the proper factor taken from the curve shown in Fig. 15. The product should be either added to or subtracted from the weight assumed, in order to determine the correct truss weight.

As a check on the validity of this method, consider the case of a 400-ft, simple-truss bridge of 45-ft clear roadway, in silicon steel, for which Fig. 1 (*b*) gives 20 as the percentage ratio. The make-up of total vertical load (in pounds per linear foot) is as follows:

Live load plus impact	3 440	
Flooring	5 470	
Hand-rails	120	
Floor system	1 575	
Lateral system	740	
Trusses (assumed)	2 600	
Total vertical load	13 945	
The percentage ratio of 20 gives		2 789
Assumption		2 600
Difference		189

For a percentage ratio of 20, Fig. 15 gives a multiplier of 1.25, which yields the correction: 189 × 1.25 = 236; and the correct truss weight, 2 600 + 236 = 2 836. With this truss weight, the new total vertical load will be 14 181. Applying the percentage ratio of 20, $W = 2\,836$, which checks.

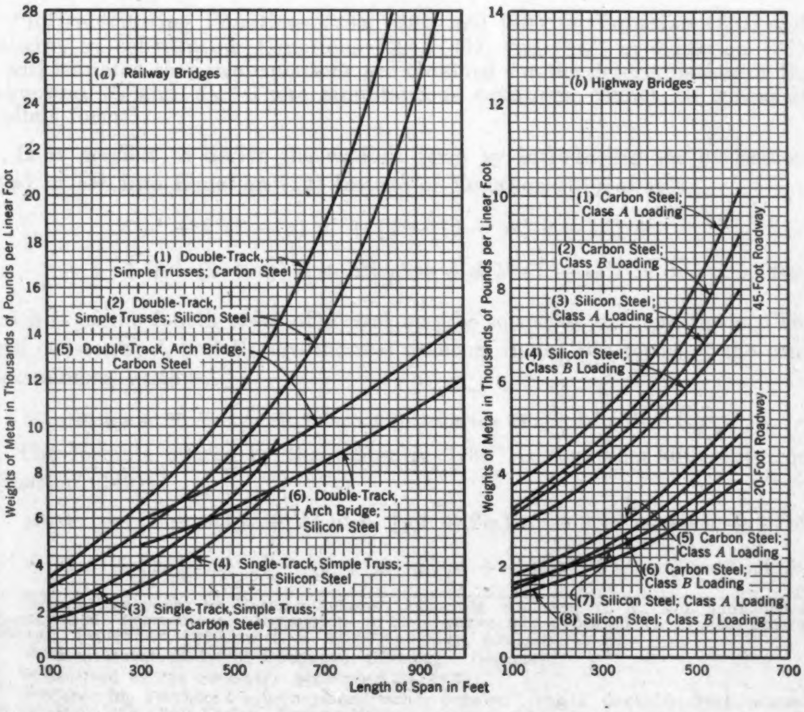

FIG. 3.—TOTAL WEIGHTS OF METAL PER LINEAR FOOT OF SPAN, IN ARCHES AND SIMPLE TRUSS SPANS.

Had the assumption been 3 000 lb per lin ft, the total vertical load would have been 14 345 lb per lin ft. Applying a percentage ratio of 20, $W = 2\,869$; a value of 3 000 was assumed, indicating a difference of 131. Multiplying 131 by 1.25 yields a correction of 164 which, subtracted from 3 000, leaves 2 836 — a check on the computation. This method is generally much quicker than the one involving a series of guesses and substitutions.

In conclusion, the writer desires to express to the gentlemen who have done him the honor to discuss this paper (both individually and collectively) his deep appreciation of their courtesy. This applies particularly to the generous comments by Messrs. Shaw, Jonah, Thomson, and Hanna, who approve the paper wholeheartedly.

Thanks to Mr. Downey, the writer has discovered a mistake in Fig. 3 (*b*). Fortunately, it is of no real importance because all the weights in Fig. 3 are intended only for the tyro, and by him to be used solely in making his first assumption of metal weight when finding the trial total load per linear foot. The effects of any incorrectness in the assumed weight will be entirely cancelled by the first-made trial computation. However, that is no valid reason for allowing an inaccuracy to remain in the paper; hence Fig. 3 has been revised as shown herewith.[32a]

[32a] To replace the present Fig. 3 in *Transactions*.

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DISCUSSIONS

LINE LOAD ACTION ON THIN CYLINDRICAL SHELLS

Discussion

BY MESSRS. W. FLUGGE, AND ANTON TEDESKO

DR. W. FLÜGGE[15] (by letter).[15a]—In general, the computations necessary for a numerical solution of shell problems have heretofore been very cumbersome. In the special case of a cylindrical shell of long span these computations may be simplified by applying the principles developed by J. W. Geckeler[6] and the author is to be congratulated on his analysis of the problem.

Following Equation (44) the author states:

"By substituting Equations (38), (39), and (40), in Equation (35), and, likewise, by substituting Equations (42), (43), and (44), in Equation (36), it will be noted that two equations are obtained for the determination of the unknowns, J_2 and K_2. These simultaneous equations cannot be solved in explicit form."

It is possible to derive the explicit form by introducing the expression, $F(\phi) = e^{\mu\phi}$ into Equation (29) instead of Equation (31), thus[16]:

$$\mu^8 + 2\,(1 - a)\,\mu^6 + [1 - (4 + m)\,a + a^2]\,\mu^4 + [-\,(2 + m)\,a + a^2]\,\mu^2 + 4\,a^2 b^2\,(1 - m^2) = 0 \ldots\ldots (149)$$

which is of the fourth order of μ^2 and may be solved by exact methods. The shell constants, J_1 and K_1, are the real numbers in the two terms of the complex number, μ; thus,

$$\mu_{1,2,3,4} = \pm\,(J_1 \pm i\,K_1), \text{ and } \mu_{5,6,7,8} = \pm\,(J'_1 \pm i\,K'_1)$$

The formula offered by the writer in 1934[16] is somewhat different from Equation (149):

$$\mu^8 + [-\,(2 + m)\,a + 2\,]\,\mu^6 + [\,(1 + 2m)\,a^2 - 2\,(2 + m)\,a + 1\,]\,\mu^4 + [-\,m\,a^3 + (1 + m)^2\,a^2 - (2 + m)\,a\,]\,\mu^2 + 4\,a^2\,b^2\,(1 - m^2) = 0 \,..(150)$$

NOTE.—The paper by Herman Schorer, Assoc. M. Am. Soc. C. E., was published in March, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1935, by I. K. Silverman, Jun. Am. Soc. C. E.

[15] Privatdozent, Univ. of Göttingen, Göttingen, Germany.

[15a] Received by the Secretary September 23, 1935.

[6] "Ueber die Festigkeit achsensymmetrischer Schalen", von J. Geckeler, *Forschungsarbeiten*, Heft 276, VDI-Verlag, Berlin, 1926.

[16] "Statik und Dynamik der Schalen", von W. Flügge, Berlin, 1934, p. 139.

The disparity is of no numerical importance, however, and is to be explained by the different treatment of some second-order terms in the fundamental equations of the shell theory. It disappears entirely if Poisson's ratio, *m*, is made equal to zero, an assumption that may simplify the numerical computations, particularly of reinforced concrete shells. The error due to this assumption will be less than that introduced by the basic simplifications of the paper.

ANTON TEDESKO,[17] ESQ. (by letter).[17a]—In presenting a rather complex subject in a clear and useful form, Mr. Schorer has performed a splendid service to the profession. The suggested method gives good approximations of practical value over a wide range of cases. It would be interesting to investigate its degree of exactness beyond the limits given by the author.

Working for Dr. Finsterwalder, the writer has used other practical simplifications of his theory, the results of which checked well with those of the accurate method. Deformations may be calculated for statically determinate membrane stress conditions, such as are impossible in manifold statically indeterminate systems. Consequently, these deformations are eliminated in the calculations for the latter system. The unknown and resultant stress values of the indeterminate system are obtained from elastic equations conforming to the edge conditions. Where the differences of numerical values of trigonometric functions of the angle, ϕ, give inexact results, the equations for deformations are expressed in power series with the angle, ϕ, as the argument. In these, and in his trigonometric series, Dr. Finsterwalder neglected terms of higher order. He decreased the degree of statical indeterminacy by pairing a vertical and a horizontal force component with a moment, M_2, in such a manner that full restraint is obtained at the edge; or, in other words, that the direction of the tangent of the cross-sectional curve at the edge remained fixed. In the second series of calculations the tangent is assumed to rotate freely. After the degree of actual restraint is determined, the values already obtained are re-adjusted accordingly. When considering roof shell problems the value, N_2, at the edge can often be neglected because of its rather small size as compared with loads acting on the edge.

The author's corrected values, J_2, K_2, J'_2, and K'_2, as given in his Equations (70), (71), (72), and (73), should be used successfully in Finsterwalder's accurate method.

A very clear and almost popular explanation of the edge problem was given by H. Rüsch,[18] whose name second to that of Finsterwalder is connected with the development of the bending theory of cylindrical shells. Valuable bibliography on the subject has been listed by Mr. A. Floris in his discussion[19] of a paper by D. C. Coyle, M. Am. Soc. C. E. The fundamental equations and principles of the author's paper were given and discussed by H. Savage.[20]

[17] Structural Engr., Roberts & Schaefer Co., Chicago, Ill.

[17a] Received by the Secretary September 24, 1935.

[18] "Theorie der Querversteiften Zylinderschalen für schmale, unsymmetrische Kreissegmente," von H. Rüsch, pub. by R. Noske, Borna-Leipzig, 1931.

[19] *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 1173.

[20] *Concrete and Constructional Engineering*, Vol. XXV, 1930, p. 490.

Using his accurate theory Dr. Finsterwalder[21] has computed the stress values, T_1, S, T_2, and M_2 for the roof of the Central Market, in Budapest,[22] Hungary. The roof is constructed of cylindrical barrels of a span, $L = 136$ ft, a width of 38 ft 9 in., and a radius, $R = 32$ ft 10 in. It consists of segmental shells in combination with edge members, the cross-section of which

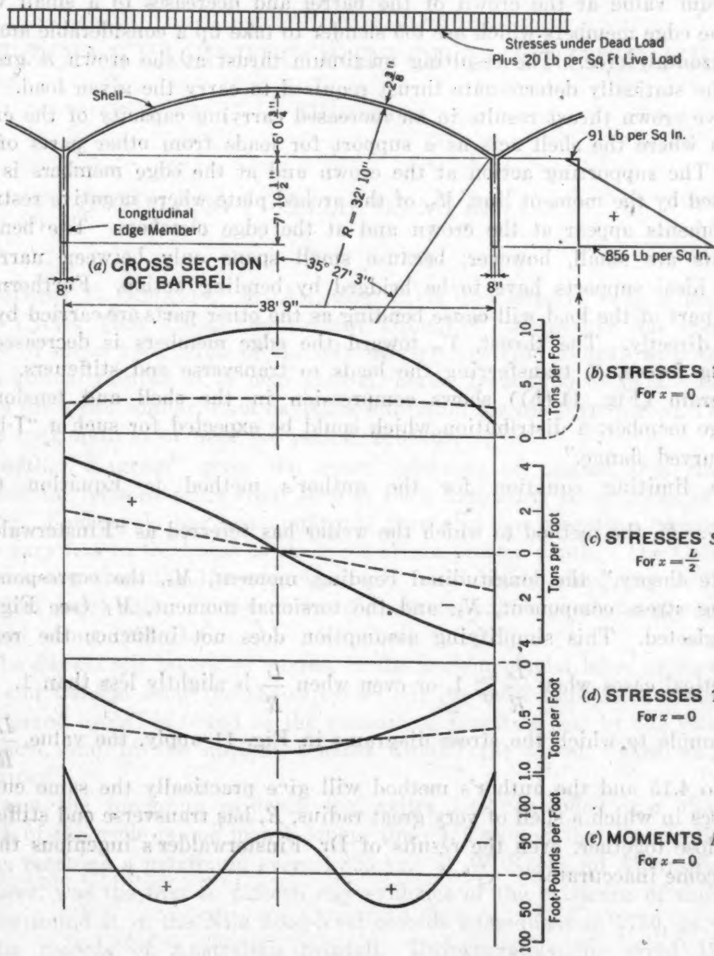

FIG. 11.—STRESS COMPONENTS IN BARREL SHELL.

is shown in Fig. 11. A graphic picture of the meaning of the shell stress components, as discussed by the author, is given in Fig. 11 (*b*), 11 (*c*), 11 (*d*), and 11 (*e*). The stresses are given for $x = 0$ and $x = \frac{L}{2}$, with reference to Fig. 1 of the paper.

[21] *Ingenieur Archiv*, Vol. IV, 1933, p. 57.

[22] *The Architectural Forum*, Vol. LX, May, 1934, p. 354.

The thrust, T_2, in the transverse direction of the shell, according to Equation (101) for the membrane system, is a direct function of the radius and the load of the shell. These values of T_2 are indicated by a broken line in Fig. 11 and are nearly constant over the width of the barrel. The thrust, T_2, of the statically indeterminate system is indicated by a solid line. It has its maximum value at the crown of the barrel and decreases to a small value near the edge members which are too slender to take up a considerable amount of horizontal force. The resulting maximum thrust at the crown is greater than the statically determinate thrust required to carry the given load. The excessive crown thrust results in an increased carrying capacity of the crown portion where the shell acts as a support for loads from other parts of the shell. The supporting action at the crown and at the edge members is also expressed by the moment line, M_2, of the arched plate where negative restraining moments appear at the crown and at the edge members. The bending moments are small, however, because small spans only between narrowly spaced ideal supports have to be bridged by bending action. Furthermore, only a part of the load will cause bending as the other parts are carried by the thrust directly. The thrust, T_2, toward the edge members is decreased by shearing forces, S, transferring the loads to transverse end stiffeners. The T_1-diagram (Fig. 11 (*b*)) shows compression in the shell and tension in the edge member, a distribution which could be expected for such a "T-beam with curved flange."

The limiting equation for the author's method is Equation (46), $\frac{L}{R} \geqq \pi$. In the method to which the writer has referred as "Finsterwalder's accurate theory," the longitudinal bending moment, M_1, the corresponding shearing stress component, N_1, and the torsional moment, M_t (see Fig. 3), are neglected. This simplifying assumption does not influence the results in practical cases when $\frac{L}{R} \geqq 1$, or even when $\frac{L}{R}$ is slightly less than 1. For the example to which the stress diagrams in Fig. 11 apply, the value, $\frac{L}{R}$, is equal to 4.15 and the author's method will give practically the same curves. For cases in which a shell of very great radius, R, has transverse end stiffeners quite close together, even the results of Dr. Finsterwalder's ingenious theory will become inaccurate.

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DISCUSSIONS

FLOOD-STAGE RECORDS OF THE RIVER NILE

Discussion

BY HALBERT P. GILLETTE, M. AM. SOC. C. E.

HALBERT P. GILLETTE,[18] M. AM. SOC. C. E. (by letter).[18a]—In a paper on "The Cycles That Cause the Present Drought"[19] the writer has compared curves of mean flood levels of the Nile River from 1735 to 1919 with those of mean annual rainfall at or near Boston, Mass., from 1750 to 1934. The two curves are quite similar, each having a minimum for the 5-yr. period, 1790–94, and a maximum at or near the period, 1865–69.

Another diagram[20] gives the mean thickness of annual tree rings by decades, for Arizona pines from 1390 to 1909, indicating three cycles of about 152 yr. Each cycle has less amplitude than the one preceding because tree rings vary less in thickness as the roots attain greater depth. The California sequoias also show the 152-yr cycle clearly, particularly when they are young. The first great valley in the curve of their annual rings shows a maximum drought about 1255 B. C.

The annual silt layers, or varves, in the ancient glacial lakes of New England and Canada show the 152-yr cycle; but the most impressive evidence of this "grand cycle" is found in the recessional moraines left by the retreating ice sheet, and in the ancient beaches around the Great Lakes and Lake Winnipeg.

From the foregoing evidence the writer has concluded that the exact length of the cycle is one month longer than 152 yr and that its amplitude is cyclic, reaching a maximum every 1 825 yr. T. W. Keele, an Australian civil engineer, was the first to discern any evidence of the existence of the cycle[21] and he found it in the Nile flood-level records subsequent to 1736, as well as in the records of Australian rainfall. Unfortunately, he erred 12% in determining its length.

NOTE.—The paper by C. S. Jarvis, M. Am. Soc. C. E. was published in August, 1935, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[18] Editor, *Roads and Streets*, Chicago, Ill.

[18a] Received by the Secretary October 5, 1935.

[19] *Water Works and Sewerage*, August, 1935, p. 289.

[20] *Loc. cit.*, Fig. 2, p. 290.

[21] "The Great Weather Cycle," by T. W. Keele, *Proceedings*, Royal Soc. of New South Wales, Vol. 44, 1910, p. 25.

By means of tree rings the writer estimates that the year of maximum drought due to this cycle will be 1939; but since there are a dozen other cycles longer than 2 yr, not to mention several shorter than that, there may be years even dryer than 1939, in the near future. This is illustrated by reference to Table 2,[22] in which the mean amplitude (Column (2)) is the percentage departure of the cyclograph peak or valley from the mean. The cyclographs were those derived from Arizona pines (1391–1910) except for the $100\frac{2}{3}$-yr cycle, for which early sequoias were used. The sequoias usually have amplitudes about two-thirds as great as Arizona pines; therefore, the 13% in Column (2), Table 2, should be increased about one-half. The amplitude varies periodically through the cycle in Column (3), Table 2; and Column (4) gives a year in which a peak of the basic cyclograph had its greatest amplitude, which is to be used as an epoch date.

TABLE 2.—THIRTEEN RAINFALL CYCLES

Basic cycle, in years	Amplitude: Mean (percentage)	Amplitude: Cycle, in years	Year (A. D.) of maximum rainfall	Basic cycle, in years	Amplitude: Mean (percentage)	Amplitude: Cycle, in years	Year (A. D.) of maximum rainfall	Basic cycle, in years	Amplitude: Mean (percentage)	Amplitude: Cycle, in years	Year (A. D.) of maximum rainfall
(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
$1\frac{17}{18}$	4	35	1911	$13\frac{11}{12}$	8	167	1816	$48\frac{5}{6}$	12	293	1756
$3\frac{1}{6}$	6	19	1900	$18\frac{5}{6}$	11	113	1889	$69\frac{2}{3}$	15	209	1760
$4\frac{1}{9}$	7	37	1922	$23\frac{5}{6}$	12	143	1859	$100\frac{2}{3}$	13	302	1912
$5\frac{13}{18}$	6	103	1919	$39\frac{2}{3}$	10	119	1921	$152\frac{1}{12}$	25	1 825	1711
$7\frac{5}{9}$	6	68	1921	.....	.....	.....	.....	.....	.....	.....	.....

Four principles seem to be indicated by Table 2, in its relation to the paper by Mr. Jarvis, namely, (1) the amplitude of a rainfall cycle tends to increase with the length of the cycle; (2) the amplitude itself is cyclic; (3) cycle lengths approximate a geometrical progression series, with a ratio equal to $\sqrt{2}$; and (4) cycle lengths are either an integral number of months, or of two-thirds of a month.

Since sequoia tree rings have been measured spanning 3 200 consecutive years, it follows that they supply a means of determining lengths of cycles with great precision. A total of one hundred and thirty-three 24-yr cycles can be traced by the sequoia data. Therefore, if the peak of the cycle is determined near the beginning and near the end of the 3 200 yr, the error in the length of the cycle cannot exceed $\frac{1}{130}$ yr.

For the foregoing reasons the writer finds that tree rings are preferable to level records of the River Nile in deriving the lengths of cycles. Further-

[22] *Water Works and Sewerage*, August, 1935, p. 292.

more, the low-water levels of this river are more variable than the flood-stage records cited by Mr. Jarvis and, therefore, are preferable for cycle analysis. A simple method of cycle analysis has been recommended by the writer[23] which is similar to that devised by Balfour Stewart.[24] It should be noted that this method or Schuster's[25] harmonic analysis modification of it will yield pseudo-cycles of two-thirds, one-half, one-third, or one-fourth the length of the true cycle. No writer has previously called attention to this fact. It arises from the shape of the rainfall curve of a cycle; and, consequently, meteorological literature abounds in pseudo-cycles.

Except for the records of the River Nile, man-kept records by weather cover such few years that they are of little value in determining cycles longer than about 6 yr. It often requires data covering more than 30 cycles to establish the existence of a cycle with a high degree of probability, because there are so many cycles that they tend to hide each other. Two rainfall cycles (approximately 152 yr and 101 yr) often are very clearly evident in spite of the other obscuring cycles. Occasionally, at intervals of 35 yr, the cycle of approximately 2 yr is well defined in winter temperatures and in laminated silt or varves.[26]

[23] *Water Works and Sewerage*, August, 1935, p. 289.

[24] "World Weather," by H. H. Clayton, p. 376.

[25] "The Periodogram of Magnetic Declination," by Arthur Schuster, *Transactions*, Cambridge Philosophical Soc., Vol. XVIII (1899), p. 107.

[26] "Recession of Last Ice Sheet in New England," by Ernst Antevs, Plates I to V, p. 121 *et seq.*

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DISCUSSIONS

DISTRIBUTION OF STRESSES UNDER A FOUNDATION

Discussion

BY MESSRS. MARSHALL G. FINDLEY, AND M. A. BIOT

MARSHALL G. FINDLEY,[32] ASSOC. M. AM. SOC. C. E. (by letter).[32a]—The problem discussed by Mr. Cummings is one of a group, which may be suggestively subdivided as follows:

1.—Footings in which one plane section only may be analyzed: (*a*) Wall footings, centered (the simplest case); (*b*) flume footings (rectangular slabs with parallel walls along two opposite edges); and (*c*) continuous footings under a series of parallel walls.

2.—Simple footings symmetrical around a central point: (*a*) Square footings with central load; (*b*) circular shaft footings (chimneys, etc.); and (*c*) square shaft footings.

3.—Eccentric footings: (*a*) Square footings with eccentric load; (*b*) building column footings near property lines; (*c*) retaining wall footings; (*d*) footings for crane columns and movable bridges; and (*e*) base slabs of concrete reservoirs.

In all these types of footings, in the usual varieties of soil, actual pressures are unquestionably small at the edges of the footings, and greatest under or near heavy load concentrations. The usual assumption of design is that under a complex footing, earth stresses vary uniformly from one edge to the other. This usual assumption is clearly just as inaccurate in the case of complex footings as it is in simple cases, such as Footings 1(*a*) and 2(*a*). Furthermore, this inaccuracy is greater in some types of soil than in others.

In consideration of footing problems, two points of view have arisen. One is that of soil mechanics, or the analysis of conditions in the soil under the structure. From this point of view the assumption of uniform or uniformly varying pressure at the plane of contact is often actually unsafe. In certain types of clay, for instance, it is now known that a large footing sometimes

NOTE.—The paper by A. E. Cummings, Assoc. M. Am. Soc. C. E., was published in August, 1935 *Proceedings*. Discussion on this paper has appeared in *Proceedings* as follows: October, 1935, by Messrs. Clement C. Williams, D. P. Krynine, and L. C. Wilcoxen.

[32] Structural Designer, Water Purification Plant, Milwaukee, Wis.

[32a] Received by the Secretary October 24, 1935.

cannot carry proportionately as large a load as a small footing; the greater building up of pressure in the center may start a crushing action of particles of clay holding water in integral union, which in time will lead to the settlement of the larger footing.

The second point of view is that of the designer of the footing slab. From his point of view, the assumption of uniform or uniformly varying pressure is almost always somewhat on the safe side. Usually the loss of economy is not very serious. In the unusual conditions that occur occasionally in which this assumption is very extravagant there is little published precedent as to the proper method of slab design.

The author's review of conditions beneath a simple footing should be studied carefully not only by students of soil mechanics, but also by designers of foundation slabs. Strangely enough, these two points of view are not actually co-ordinated as well as they might be.

In the design of footing slabs the exact theoretical formula is often not of great importance. For instance, in the case of a simple wall footing, a sine formula seems to be theoretically correct in stiff homogeneous material for distribution of pressure at the plane of contact; but considering the resulting design of footing, it may not make any appreciable difference whether a sinoid, parabolic, or an elliptic distribution curve of footing pressure is assumed. The important point is to take account of the deflection of the cantilever edges; this may be done even by means of a very rough, straight-line diagram, without serious error in shear or moment diagrams.

Such rough straight-line diagrams may be used with profit in the design of certain more complex types of footings. It is not the intention to cast any doubt on the advisability of theoretical and experimental work such as that cited by Mr. Cummings; but there are certain types of footings, some of which are among those outlined herein, in which even with rough straight-line assumptions as to footing pressures, design becomes very complex mathematically. The exact conditions are not known through experiment; nor are they accessible to exact scientific imagination; the soil is not perfectly homogeneous, although on the whole it is substantial and dependable; and the final conditions as to ground-water are not determined or are not constant. Under such conditions, it seems more in line with exact scientific procedure to make a rough but probable assumption and then to compute, exactly, certain of its consequences, than to use the very improbable assumption of uniformly varying load. Furthermore, the less probable assumption occasionally results in design that is economically impossible, and obviously unnecessary. Painstaking detailed investigation has clearly proved certain general truths about foundation pressure distribution. It is possible to use these general truths in a rough approximate way, under conditions where detailed application is impossible.

M. A. Biot,[33] Esq. (by letter)[33a]—The classical theory of elasticity is based on a linear relation between stress and strain. This fact introduces an

[33] Univ. of Louvain, Louvain, Belgium.

[33a] Received by the Secretary October 21, 1935.

important simplification in mathematical investigations in that the superposition principle can be used and the summation of elementary solutions leads to other solutions.

It is important to note that for a material that does not satisfy Hooke's law, this superposition principle does not hold and, in general, it will not be permissible to add up solutions due to loads acting independently in order to find the stresses due to their combined action.

In the case of certain materials like sand, however, the pressure distribution is assumed to be given by a semi-empirical law more general than that of Boussinesq formulated by Equation (3) of the paper. In case the pressure distribution does not coincide with that of Boussinesq, this must be due to the fact that the material is either not isotropic or does not satisfy Hooke's law. The latter is true, for instance, if some kind of gliding occurs inside the material. Obviously, then, it is not permissible to superpose solutions of the form of Equation (3) in order to find the effect of distributed loads.

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DISCUSSIONS

SOME LOW-TEMPERATURE CHARACTERISTICS OF BITUMINOUS PAVING COMPOSITIONS

Discussion

BY MESSRS. PHILIP W. HENRY, J. T. L. MCNEW, AND ROY M. GREEN

PHILIP W. HENRY,[6] M. AM. SOC. C. E. (by letter).[6a]—Although the author may be justified in stating (see "Synopsis") that "little, if any, study has been made concerning low-temperature performance in paving compositions" and (see "Introduction") that "scientific data have not been available with reference to the characteristics of bituminous paving compositions over a wide range of temperatures, particularly low temperatures," nevertheless, as early as 1887, it was the practice to use a softer asphalt cement in such cities as Buffalo, N. Y., and Omaha, Nebr., where asphalt pavements laid a few years earlier had cracked badly. In 1887 the Company with which the writer was connected used a softer asphalt cement without any change in the mineral aggregate or percentage of bitumen to meet this situation. When the writer became Superintendent of the Company in Omaha, in 1889, he continued the practice, with the result that although the asphalt pavements laid a few years previously showed the worst cracking that he ever beheld before or since, all the pavements laid with the softer asphalt cement came through the winters without cracking. It was also the practice to use a still softer asphalt cement on streets of light traffic.

In those days no attention was paid to the mineral aggregate, and little to the percentage of bitumen. In all cities practically the same number of pounds of sand, stone dust, and asphalt cement were used in the batch, regardless of the grading of the sand. Fortunately, in most cities, suitable sand was at hand, although in a few cities, St. Louis, Mo., particularly, the sand then used was deficient in fine material (as was discovered later), and the pavements lacked the durability which pertained to most of the asphalt pavements of those days.

Until 1888 there was no method of recording the consistency of a given asphalt cement. Previous to that time the consistency was determined by

NOTE.—The paper by Hugh W. Skidmore, Assoc. M. Am. Soc. C. E., was published in August, 1935, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[6] Cons. Engr., New York, N. Y.

[6a] Received by the Secretary September 23, 1935.

chewing the cement, and the foremen of the asphalt plants became so expert that they could determine in this way the penetration of a given asphalt cement within four or five points.

Mr. Clifford Richardson, the author of the first authoritative publication concerning all phases of an asphalt pavement, was well aware of the importance of meeting low temperatures with a softer asphalt cement. In discussing certain pavements laid in 1897 he wrote[7]:

"Experience has shown, however, that in these surfaces, although the average percentage of bitumen reached 10.5, it was in most cases too hard, averaging 58, which has resulted in some cracking. In subsequent years, therefore, it has been the practice to use softer asphalt cement. The results have been very satisfactory."

In another place he states[8]:

"Cracks of the second description, due to the use of asphalt cement which is too hard or which has become hardened by being mixed with too hot sand, or to this cause combined with others, are the form which is most commonly met with * * *. Such cracks are due to the fact that the hard asphalt is too brittle at low temperatures to yield to the contraction of the surface. It fractures under the tensile stress imposed upon it. * * * Cracks which are due to the fact that the mixture is deficient in bitumen, in consequence of which the surface does not possess sufficient tensile strength, regardless of ductility, at low winter temperatures, are not as frequent as those due to a hard bitumen, since in such a mixture, disintegration with the formation of holes takes place, as a rule, before cracking."

Notwithstanding this early general knowledge, arrived at through costly experience (in those days, asphalt pavements were guaranteed from 5 to 20 yr), the data furnished by the exhaustive study made by Mr. Skidmore would seem to establish a scientific basis for designing asphalt pavements to meet the low winter temperatures.

Incidentally, it is curious to note that there has been practically no change in the method of laying asphalt pavements on city streets in the past fifty years and more. In the Eighties the same types of tampers, smoothers, fire-wagons, and rollers were used as are used to-day. This is in contrast to the laying of the concrete base, which was then mixed entirely by hand.

J. T. L. McNew,[9] M. Am. Soc. C. E. (by letter).[9a]—It is indeed gratifying that Mr. Skidmore has presented the results of this particular investigation. Undoubtedly, he has treated a phase of the subject which is very important, and one, perhaps, which has been neglected too long.

Notwithstanding the fact that technologists have always admitted that asphalt mixtures have no appreciable strength in flexure, the tendency of many engineers in practice has been to build as much beam strength in the mixes as possible, with the result that flexibility, self-healing, and shock-resisting properties were sacrificed. This tendency is indicated by specifications which provide for extremely high filler content, low bitumen content, and harder

[7] "The Modern Asphalt Pavement", Second Edition, p. 321.

[8] *Loc. cit.*, pp. 480–483.

[9] Prof. of Highway Eng., Agri. and Mech. Coll. of Texas, College Station, Tex.

[9a] Received by the Secretary, October 1, 1935.

asphalt. Part of this tendency toward so-called highly stable mixtures has been due to a misconception as to what construction methods are feasible for field use. Mixtures that are compressible under laboratory conditions may be entirely unworkable on the road or on the street. When such conditions are encountered the novice resorts to hotter mixtures and still less bitumen, all of which tends toward making a bad mixture worse.

The effect of the mixing temperature on the bituminous cement is of considerable importance, especially since many of the mixtures classified as highly stable are also low in aggregate voids and, in turn, low in total bitumen-carrying capacity. Reference to analyses of samples from existing pavements show conclusively that plant heat and agitation during mixing may lower the penetration of the asphalt as well as the percentage of bitumen so much as to destroy the self-healing properties entirely. As early as 1919, Roy M. Green, M. Am. Soc. C. E.,[10] reported numerous examples of pavements in which the aggregate was bonded with asphalt from 10 to 20 penetration and yet it was known that the material used was not as hard as the analyses indicated. These extremely low penetrations could not be chargeable entirely to a natural loss of the more volatile fractions by weathering.

The writer has always been unable to find much good in the theory requiring the use of harder asphalts for increasing stability as measured by shearing tests. Shearing displacements are most likely to occur during periods of high temperature; and yet when the air temperature is 100° F the pavement temperature will often be 145° F, or so hot that any usable steam-refined asphalt cement will be in a liquid, or a very soft, condition. Obviously, the cementitiousness of two similar residues having slightly different degrees of fluidity cannot be measured as a large quantity. If most asphalt binders are in reality liquids or near liquids at summer pavement temperatures, then perhaps the real question to be answered is: What mixture design will serve both the summer conditions as well as the winter extremes? The author's investigations are interesting in that the results give some information on that question and cast suspicion on the theory sometimes advanced that harder asphalt cements produce better pavements.

If it is assumed that an asphalt surface is on an adequate foundation, low temperature failures of the surface are likely to occur as a result of mixtures that are too lean in asphalt cement, as a result of brittleness of the binder, or by raveling of the surface. Winter raveling may be due to the brittleness of the binder or it may be caused by freezing of impregnating water in the open mixtures. Brittleness in a pavement probably is more easily measured by rapidly applied loads than by slowly applied increments, and such a thought gives rise to the question: Why should not all stability tests on asphalt mixtures be made either with rapidly increasing loads or with loads applied suddenly? Regardless of what stability test is used on a bituminous paving mixture, the strength increases as the percentage of bituminous binder increases from a decided deficiency to some percentage that provides a maximum strength. After the maximum strength is reached, if the bituminous

[10] Bituminous Pavement Investigations, Pt. I and Pt. II, Texas Eng. Experiment Station, 1919 and 1921.

cement is further increased, the strength of the mixture decreases and continues to decrease upon the addition of bitumen until the samples are made of a preponderance of bituminous cement and little or no aggregate. The difficulty encountered in correlating the results of the different types of tests lies in the fact that the maximum strength does not occur for the same percentage of bitumen; for example, one testing procedure may indicate a maximum strength at about 8% bitumen, whereas another type of test will probably show a maximum strength for as high as 9½ per cent.

For a number of years the writer has been using a modified shear test made by forcing a frustum of a cone out of a cylindrical specimen of pavement under a gradually applied load, and has been able to verify conclusions of other investigators who used somewhat different tests to study the effects of different variables on the strength of mixtures. He has never been completely satisfied with the test as performed, however, because of the conviction that the manner of applying the load does not simulate the load application made by traffic; nor has he been able to propose such a loading device that is entirely satisfactory. Even the most rigid or stable asphalt pavements, as measured by the shearing strength tests, are in reality plastic mixtures that undergo movement on the street; and such movement does, to an appreciable extent, cause an internal re-arrangement of particles. Plasticity, then, is a property of a bituminous mixture that must be admitted and the most made of it as a quality of the mixture, because it cannot be eliminated entirely. A bituminous mixture may be plastic enough to flow gradually when subjected to a load of any kind for a long time and yet if the same mixture were loaded suddenly either in shear, compression, tension, or flexure, the plastic properties would not prevent failure.

As stated by the author, the ductility of the bituminous cement at low temperatures is of great importance. For that matter, any test for consistency at low temperatures is of value because plasticity is the desirable quality rather than brittleness in the lower ranges of temperature. It appears that the mixture which may be judged the strongest in shear at 140° F may be decidedly too brittle at 4° or 5° C.

In 1929, the writer undertook the direction of some research work on bituminous paving mixtures in the hope of developing certain modifications of test procedure that would detect properties of brittleness. Specimens were tested first by determining the intensity of load, applied gradually, that would be required to push a frustum of a cone out of a specimen 5 in. in diameter and 2 in. thick. From this test a strength curve could be plotted to show the effect of varying the bitumen. Later, another series of samples like the first was tested by knocking out the frustum of a cone of the same size by means of a hammer falling from a constant height. In this latter case the number of blows to cause failure was plotted as the ordinate against the percentage of bitumen as the abscissa. As was to be expected, the general shape of the strength bitumen curve was the same for the two tests, but the percentage of bitumen required for maximum strength was more for the impact test than for the shear test. In other words, the samples that were strongest under shear were not the strongest under impact. Of course, no one has

proposed that the mixture which is strongest in shear should necessarily be the one chosen for the street; however, it has been a debatable question as to what variation from maximum results should be specified for the paving job. The writer is convinced that the foundation of a road must be able to withstand the traffic load before any surface will be capable of fulfilling its mission. If such a foundation is adequate, the function of the surface is to prevent wear, absorb impact, distribute load, and, in other ways, preserve the life of the load-carrying medium, regardless of weather. To fulfill that function the surface must be plastic enough to be either flexible or self-healing, stable enough to prevent waving, and designed so that it may be controlled as to uniformity in all the construction operations.

Since ground movement is also prevalent in winter months there appears to be every evidence to indicate that flexibility must be a quality of bituminous surfaces during winter extremes of temperature. It appears also that the mixture design, as well as the choice of all materials, must be predicated upon a proper evaluation of all demands to be made upon a surface, and not on the extremes of summer temperatures alone.

Roy M. Green,[11] M. Am. Soc. C. E. (by letter).[11a]—Engineers and asphalt technologists are indebted to the author for this contribution toward placing the design and performance of asphaltic mixtures upon a more sound and rational basis. He is to be particularly commended for his demonstration of the following facts: (1) A desirable design of an asphaltic mixture for cold climates is one in which the asphalt cement slightly over-fills the voids in the mineral; (2) in attempting to obtain high stability in a mixture there is grave danger of using a filler content which is too high; and (3) asphalt cements of higher penetration are desirable in the colder climates.

A reading of this paper and a study of the curves in Fig. 2 leaves the impression that the California cements (Specimens 1, 2, and 3) are undesirable in cooler climates. This is not a stated conclusion of the author, however. In the Cities of Omaha and Lincoln, Nebr., there are a number of sheet asphalt pavements which have been in service since 1905 and before. These pavements were laid with California cements, and have withstood the winter weather of this section of the country without the least distress and are in excellent condition to-day. However, the mixtures are rich and contain asphalt cement in excess of that necessary to fill the voids in the mineral.

It would have been very enlightening had the author made further mixtures with these cements, carrying the asphalt cement to 10% above normal, as he did the mixtures given in Fig. 5. Would this added asphalt cement produce a mixture such that the curve would not reverse itself at the lower temperatures? Is it possible, to a certain extent, to improve upon the performance of a given asphalt cement by altering the composition of the mixture from the standard conditions set up by the author's experiments?

[11] Pres. and Mgr., Western Laboratories, Inc., Lincoln, Nebr.

[11a] Received by the Secretary October 9, 1935.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

FAILURE THEORIES OF MATERIALS SUBJECTED TO COMBINED STRESSES

Discussion

BY MESSRS. W. P. ROOP, AND H. F. MOORE

W. P. ROOP,[18] M. AM. SOC. C. E. (by letter).[18a]—The need for a "failure theory" is not immediately apparent; it is a rather naive but widespread idea that a safety factor, taken as the ratio of ultimate to working stress, represents the margin provided to prevent failure in spite of the designer's mistakes. In point of fact "there is a considerable number of variables outside of magnitude of stress, involved in the problem of strength"[19]; the burden thrown on the safety factor by uncertainty as to what determines failure under combined stresses is not responsible for a large share of the margins of strength found necessary.

Nevertheless, some sort of criterion by which to allow for combination of stress is necessary, and as the other variables in the problem of strength are subjected to more searching analysis, it is right that this one should also be re-examined. For this purpose sixteen different theories are reduced to forms suitable for comparison, both analytical and graphical. It is understood that the curves shown represent boundaries beyond which plotted points indicate failure. Although it is not clearly so stated, it is understood furthermore, that the co-ordinate of these points are the relative values of the principal stresses. Each such point thus represents a certain proportion between the two principal stresses, the scales being such that the ultimate strengths in simple tension and compression are correctly indicated, so that all curves (except that for Theory (8)) pass through all the four points $x = \pm 1, y = \pm 1$.

It appears in Table 1 that "failure" may mean anything from rupture to exceeding the proportional limit. In the graphic study, these differences are ignored and only the effect of combining stresses is considered, with no reference to characteristics of the material.

NOTE.—The paper by Joseph Marin, Jun. Am. Soc. C. E., was published in August, 1935 *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1935, by Messrs. J. J. Slade, Jr., T. McLean Jasper, and I. K. Silverman.

[18] Lt. Comdr. (C.C). U. S. Navy, Office of Superintending Constructor, New York Shipbuilding Co., Camden, N. J.

[18a] Received by the Secretary September 27, 1935.

[19] *Journal of Applied Mechanics*, 1935, p. 106.

The stress at a point in a homogeneous elastic medium is a condition that can be specified exactly, regardless of the combination of loads, each with its contribution to stress, by which the actual result is produced. The real questions thus appear to be: (*a*) Considering the stress at the point at which "failure" begins, what feature of the stress is it that determines the failure; or, in a word, what constitutes equivalence of stresses in causing failure? and (*b*) in strength calculations involving combined loads, how shall the effective feature of the resultant stress be estimated?

Fig. 19 indicates that the graphs generally agree in showing an elongation along the axis of half past one. Does this mean that a combination of principal stresses of the same sign is more easily withstood than a combination of opposite signs? If so, that would confirm the idea that somehow shear plays a special part in causing failure.

It appears that no more definite conclusion than this is possible from the data presented, since the spots in Fig. 19 and the data in Columns (6) and (7), of Table 1, show that the spread of observations exceeds the differences between the theories.

H. F. MOORE,[20] ESQ. (by letter).[20a]—An interesting summary of the present-day knowledge of theories of failure of materials subjected to combined stress, is presented in this paper. Table 1 is a very convenient statistical summary of the evidence. It is to be noted that, with the exception of the tests with solid cast-iron rods under combined tension and compression (at right angles), Professor Marin has used the criterion of elastic strength in all cases. He quotes the strength of a tube of cast iron as judged by "yield point". This reporting of a yield point in cast iron would seem to call for further explanation, as cast iron ordinarily does not have a yield point. At first sight, the various criteria used for elastic failure would rather confuse the evaluation of results, but a closer examination shows that for each comparison of theory with experimental results the same criterion is used, so that tests may be compared fairly satisfactorily. It would be of interest if Professor Marin had given references for the particular experiments in each case listed in Table 1, in fact his paper would be distinctly more valuable if references to original sources of data were given more fully.

Two points are not covered by the paper. The first is the failure of materials by other than elastic failure. Professor Marin has, indeed, quoted tests on cast iron in which theory was judged by the strength at fracture. These tests seem to indicate that the maximum strain theory fitted test results more closely than the other theories. This result is checked by tests obtained by Matsumura and Hamabe[21] on tests of cast iron under combined bending and torsion. In the 1933 Marburg Lecture before the American Society for Testing Materials, Dr. H. J. Gough, of the British National Physical Laboratory, quoted fatigue tests of mild steel under combined stresses[22] which indicated that, using fatigue fracture as a criterion of strength, the maximum shear

[20] Research Prof. of Eng. Materials, Univ. of Illinois, Urbana, Ill.

[20a] Received by the Secretary October 14, 1935.

[21] *Memoirs* of the Coll. of Eng., Kyoto Imperial Univ., Kyoto, Japan, February, 1915.

[22] *Proceedings*, Am. Soc. for Testing Materials, Vol. 33, Pt. II, pp. 103–104.

strain-energy theory (Theory (11) in the paper) fitted results best; and the maximum shear theory (Theory (7), in the paper) showed fairly close agreement with test results and was slightly on the "safe" side. It is quite possible that different theories hold for ductile and for brittle materials, and also for failure by fracture as opposed to failure by plastic action. This raises the question of failure by continued flow or creep, which has become important at high temperatures. No data on creep under combined stresses are available, so far as the writer knows.

Another point not covered by Professor Marin is the question of failure by "tri-axial" stress, especially a failure under equal tensile stresses along three axes of reference. So far, no effective experimental study of this phenomenon has been made. Under equal tensile stresses in three directions the shearing stress in a metal would be zero, and it is a matter of dispute whether this would strengthen the metal by preventing shearing failure, or weaken it by inhibiting the occurrence of slight localized plastic action which usually is found in metals even under light stresses, and which tends to reduce the "peaks" of localized stress which, without this plastic action, would be developed on minute areas. The writer does not know any more intriguing problem than the study of this "triple tension", which has been discussed so vigorously by Professor B. P. Haigh, of the Greenwich Naval Academy Laboratories, at Greenwich, England.

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DISCUSSIONS

THE STRESS FUNCTION AND PHOTO-ELASTICITY APPLIED TO DAMS

Discussion

BY I. K. SILVERMAN, JUN. AM. SOC. C. E.

I. K. SILVERMAN,[16] JUN. AM. SOC. C. E. (by letter).[16a]—The first solution for the stresses in a gravity dam based on the equations of the theory of elasticity was given by Lévy.[17] However, this solution holds only for an infinite wedge and does not take into account the elastic action of the base. Lévy's solution coincides with the ordinary engineering methods for the determination of the stresses in triangular dams and, at the same time, gives the law for the distribution of the shearing stresses. Until recent times very little experimental work has appeared in English or American engineering literature on this important question. The experiments[18] of Ottley, Brightmore, Wilson, and Gore made in 1907, on this type of structure, were outstanding. On the other hand, there have been many theoretical investigations and among the outstanding ones are those of Wolf,[19] Vogt,[20] and Kalman.[21] The latter investigators have shown definitely that the influence of the foundation cannot be neglected and that the stresses in the lower third of the dam are greatly affected by its elastic behavior. The author has pointed to this fact and has furnished a valuable tool in the form of the function given by Equation (50), for the determination of the qualitative effect of the restraining action of the base. This function is important since it furnishes an infinite number of free constants which may be used to satisfy any num-

NOTE.—The paper by John H. A. Brahtz, Esq., was published in September, 1935, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[16] With U. S. Bureau of Reclamation, Denver, Colo.

[16a] Received by the Secretary, September 30, 1935.

[17] *Comptes Rendus*, Vol. 127, 1898, pp. 10–15. For a complete discussion concerning the stresses in gravity dams, see "Stresses in Gravity Dams by the Principle of Least Work", by B. F. Jakobsen, and the discussion thereon, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), pp. 489–591.

[18] *Minutes of Proceedings*, Inst. C. E., Vol. CLXXII, 1907–08 Session, Pt. II, 1908.

[19] "Zur Integration der Gleichumg $\nabla^4 F = 0$ durch polynome im Falle des Staumauer Problems" *Sitzungberichte* der Kaiserlichen Akademie der Wissenschaften in Wien, 1914, Vol. 123, No. 2.

[20] "Ueber die Berechnung der Fundamentdeformation", Det Norske Videnkaps Akademi, Oslo, 1925.

[21] "Sulla validità del regime Levy nelle dighe del tipo di gravità", *L'Energia Elettica*, March and April, 1927.

ber of elastic or stress conditions. Thus, it is to be seen that the stresses may be determined to any degree of mathematical accuracy. It is to be noted that Equation (43) is a solution of Equation (3*b*) and, therefore, describes a state of stress which is in equilibrium and compatible with Hooke's law.

This very important function may be determined by the usual methods for the solution of partial differential equations when the form of the solution is indicated by the nature of the problem. Consider the case of a rectangular plate[22] with its sides parallel to the OX and OY-axes in Fig. 1. The stresses on the sides, $y = \pm c$, are described as follows:

For $y = + c$:

$$\sigma_y = - B \sin \frac{m \pi x}{l} \qquad (83a)$$

and for $y = - c$,

$$\sigma_y = - A \sin \frac{m \pi x}{l} \qquad (83b)$$

In both cases, $\tau_{xy} = 0$. From the nature of these given stresses and of the shape of the body, it is logical to assume that the Airy function is of the form:

$$F = \sin \frac{m \pi x}{l} Y \qquad (84)$$

in which Y is a function of y only. Inserting Equation (84) into Equation (3*a*) an ordinary differential equation for Y is obtained, as follows:

$$\alpha^4 Y - 2 \alpha^2 \frac{d^2 Y}{dx^2} + \frac{d^4 Y}{dx^4} = 0 \qquad (85)$$

in which $\alpha = \frac{m \pi}{l}$. The solution of Equation (85) contains four constants of integration which may be determined from the boundary conditions expressed by Equations (83) and (84). The solution of Equation (85) is:

$$Y = C_1 \sinh \alpha y + C_2 \cosh \alpha y + C_3 y \cosh \alpha y + C_4 y \sinh \alpha y \qquad (86)$$

The Airy function is:

$$F = \sin \alpha \times [C_1 \sinh \alpha y + C_2 \cosh \alpha y + C_3 y \cosh \alpha y + C_4 y \sinh \alpha y] \qquad (87)$$

Similarly, when the structure dealt with is bounded by two concentric circles or parts of concentric circles and the loads on the boundaries are expressed as functions of the angle, θ, the form of the Airy function is $F = R \sin n \theta$, or,

$$R \cos n \theta \qquad (88)$$

[22] "Theory of Elasticity", by S. Timoshenko, Eng. Societies Monographs, McGraw-Hill, 1934, p. 44.

in which R is a function of the variable, r, only. Inserting Equation (88) into Equation (3b) an ordinary differential equation is obtained for R. The solution of this equation yields the following Airy function.[23]

$$F = a_0 \log r + b_0 r^2 + c_0 r^2 \log r + d_0 r^2 \theta + \frac{a_1}{2} r \theta \sin \theta + a'_0 \theta$$
$$+ (b_1 r^3 + a'_1 r^{-1} + b'_1 r \log r) \cos \theta - \frac{c_1}{2} r \theta \cos \theta$$
$$+ (d_1 r^3 + c'_1 r^{-1} + d'_1 r \log r) \sin \theta$$
$$+ \sum_{n=2}^{\infty} (a_n r^n + b_n r^{n+2} + a'_n r^{-n} + b'_n r^{-n+2}) \cos n \theta$$
$$+ \sum_{n=2}^{\infty} (c_n r^n + d_n r^{n+2} + c'_n r^{-n} + d'_n r^{-n+2}) \sin n \theta \quad \text{.......(89)}$$

in which the constants of integration are to be determined from the boundary conditions.

The most suitable function for the stresses at a sharp corner, of the type shown in Fig. (4), is of the form:

$$F = r^{n+1} f \quad \text{........................(90)}$$

in which f is a function of the variable, θ, only. Insertion of Equation (90) into Equation (3b) gives the following ordinary differential equation for f:

$$\frac{d^4 f}{d \theta^4} + 2 (n^2 + 1) \frac{d^2 f}{d \theta^2} + (n^2 - 1)^2 f = 0 \quad \text{..........(91)}$$

The solution of Equation (91) is:

$$f = c_1 \cos (n-1)\theta + c_2 \sin (n-1)\theta + c_3 \sin (n+1)\theta + c_4 \cos (n+1)\theta . \text{(92)}$$

The constants of integration and the values of n that remain to be determined are obtained from the boundary conditions on the faces, $\theta = 0$ and $\theta = \gamma$. Expanding the terms of Equation (92), the resulting formula is identical with Equation (43) in which A, B, C, and D are constants of integration replacing c_1, c_2, c_3, and c_4 of Equation (92).

For the boundary conditions $\theta = 0$; $\sigma_\theta = 0$; $\tau_{r\theta} = 0$; and $\theta = \gamma$; $\sigma_\theta = 0$; $\tau_{r\theta} = 0$ – a set of homogeneous linear equations is obtained, namely;

$$B = 0 \quad \text{..............................(93)}$$

$$A\, n + C = 0 \quad \text{..........................(94)}$$

$$D \sin n \gamma + A \sin n \gamma \cos \gamma + C \cos n \gamma \sin \gamma = 0 \quad \text{.......(95)}$$

and,

$$D (n \cos n \gamma \sin \gamma + \sin n \gamma \cos \gamma) + A (n \cos n \gamma \cos \gamma - \sin n \gamma - \sin \gamma)$$
$$+ C (\cos n \gamma \cos \gamma - n \sin n \gamma \sin \gamma) = 0 \quad \text{..........(96)}$$

[23] "Theory of Elasticity", by S. Timoshenko, Eng. Societies Monographs, p. 114.

The simultaneous solution of Equations (93) to (96) gives only relations between the constants, namely, $B = 0$; $\frac{C}{A} = -n$; and, $\frac{D}{A} = -(\cot\gamma - n\cot n\gamma)$. Assuming that $A = -1$, then $C = n$; and $D = (\cot\gamma - n\cot n\gamma) = m$, which values are identical with those of the author. Furthermore, in order that the constants may differ from zero, the following relation must hold true:

$$n^2 \sin^2\gamma = \sin^2 n\gamma \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(97)$$

By means of Equation (97) any number of functions of Equation (43), each with a free constant, may be obtained to satisfy conditions on boundaries other than $\theta = 0$ and $\theta = \gamma$, as shown by Equation (50).

The function expressed by Equation (43) has appeared from time to time as special cases in engineering literature, but no general form has been given. H. M. Westergaard, M. Am. Soc. C. E.,[24] has applied this type of function when $\gamma = 360$ degrees. The function took the following form,

$$F = \sum_{n=1,2,3\ldots}^{n} K_n \frac{r^{n+1.5}}{2(n+0.5)(n+1.5)}\Big[(n-0.5)\sin(n+1.5)\theta - (n+1.5)\sin(n-0.5)\theta\Big]\ldots\ldots\ldots(98)$$

in which the coefficients, K_n, are free constants.

[24] "Stresses at a Crack, Size of the Crack, and the Bending of Reinforced Concrete", by H. M. Westergaard, *Journal*, Concrete Inst. November-December, 1933, p. 93.

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DISCUSSIONS

FLOOD AND EROSION CONTROL PROBLEMS AND THEIR SOLUTION

Discussion

BY MESSRS. ARTHUR G. PICKETT, AND R. W. DAVENPORT

ARTHUR G. PICKETT,[9] ASSOC. M. AM. SOC, C. E. (by letter)[9a]—In the latter half of his paper, Mr. Eaton has demonstrated conclusively that the primary cause of débris flows hazardous to communities of the Los Angeles coastal plain, is the denudation by forest fires of the brush cover of adjacent foot-hill and mountain water-sheds. These fires, if followed by the type of heavy semi-tropical rains that frequently occur in Southern California, result in sheet erosion and débris flows from the mountain sides on to the unprotected communities below. Such flows will not cease until the native brush cover has been fully restored.

No engineer familiar with the topography and geology of these watersheds, who observed the results of the La Crescenta-Montrose disaster, will question the possibility of a débris flow during such a storm, of from 50 000 to 100 000 cu yd per sq mile of water-shed burned.

Mr. Eaton presents factual data to the effect that the La Crescenta-Montrose disaster resulted in the loss of thirty lives, the destruction of 483 homes, and a total property damage of $5 000 000, all of which was the result of one storm upon a burned area of 7.5 sq miles. To this loss must be added the cost of débris removal and the necessary construction of débris basins and flood-control works adequate in size to prevent similar damage which may occur at any time in the next 5 or 10 yr, or until the native brush cover is sufficiently restored to retard excessive erosion.

Based upon data presented in the paper, the cost of débris removal following the disaster, probably exceeded $425 000, or more than $57 000 per sq mile of water-shed burned. Los Angeles County is admittedly entering a wet cycle, or period of more than average rainfall, during which several storms of greater intensity than the one which caused the disaster may be expected.

NOTE.—The paper by E. Courtland Eaton, M. Am. Soc. C. E., was published in September, 1935, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[9] Civ. Engr., Los Angeles County Surv., Los Angeles, Calif.

[9a] Received by the Secretary October 9, 1935.

Consequently, the cost of construction and maintenance of débris basins in the area during the next five years may, according to Mr. Eaton's data, exceed \$1 000 000.

The writer is thoroughly in accord with most engineers, who agree that if the mountain water-shed had not been denuded by forest fire, little or no damage would have occurred in the foot-hill communities of this flood area. It would seem, therefore, that the possibility of successful suppression of brush and forest fires adjacent to such communities should receive the primary consideration of engineers interested in flood and débris control.

Similar fires in the Hollywood Hills have been fought successfully by the Fire Department of the City of Los Angeles. R. J. Scott, Chief of the Fire Department, has found that, if trained fire fighters, properly equipped, can reach a fire within 15 min of its inception, and if 10 000 gal of water per min can be placed at their disposal for a short time, they can almost invariably suppress the fire without serious loss of water-shed cover; whereas without this water available, a loss of hundreds or even thousands of acres may be expected. Table 7 of the paper, shows that seventeen fires fought, without the advantage of an adequate water supply, resulted in a loss of 320 400 acres of water-shed in Los Angeles County, an average of 18 900 acres per fire.

Although the personnel of the Federal Forest Service and County Forestry Departments is thoroughly organized and trained, limited appropriations have prevented the installation of water systems, as well as the acquisition of other much needed facilities for forest fire protection. Until recently, such facilities have been regarded by many as an unwarranted expense, even in the relatively small areas, adjacent to heavily populated communities, where more than 90% of such fires originate.

Following the La Crescenta-Montrose disaster, plans and specifications were prepared for the construction of a project, under the work relief program, to provide a water system, motorways, firebreaks, trails, and similar facilities necessary for the fire protection of the extremely dangerous water-shed above the communities of Altadena and Pasadena.

This project was the first unit of a comprehensive plan to provide such facilities for approximately 16 000 acres of valuable but dangerous mountain water-shed, hazardous to the communities of Altadena, Pasadena, South Pasadena, Sierra Madre, Arcadia, San Marino, San Gabriel, and Rosemead, with a group population of more than 300 000.

The only feature of the plan, not included in similar projects now under construction (1935), is the water system. The quick, economical delivery of the large quantities of water required, presented many new and unusual problems which were complicated by the steep grades and topography encountered. These problems were solved, however, and an adequate water system was designed which would enable fire-fighters to reach any point in the area with heavy streams of water under gravity pressure, by the extension of not more than 500 ft of hose.

As designed, the main storage reservoir of the system is located above the highest point in the area to be protected, and will have a capacity of 1 000 000 gal. Water will be delivered to this reservoir, either from a rain-

fall catchment area, or by gravity flow through 4 000 ft of 6-in. pipe line constructed into Millard Canyon, where an adequate, reliable stream of water is always available.

From this reservoir, 6-in. steel pipe lines will carry the water down the main ridges to eight secondary reservoirs of the system, which are necessary for pressure regulation and additional storage. From these reservoirs, 6, 4, and 3-in. pipe lines will extend to roads, trails, motorways, and other strategic points throughout the area. One hundred and fifty fire hydrants, fitted with adapters to permit instant connection of either the 1½-in. hose used by the Forest Service, or the 2½-in. hose used by County Fire Departments, will be located along the various water lines.

The design will permit full delivery of water from each hydrant, even if all the other hydrants of the system are in operation at the same time. Sufficient water storage is provided to assure an adequate supply for the fighting of fire throughout the area. At points of unusual hazard, where even a city fire stream might not prove effective, small monitors or hydraulic giants have been provided to operate under higher pressures and with larger quantities of water than would otherwise be possible.

Specifications provide that all reservoirs shall be constructed in excavation, lined with reinforced concrete to prevent leakage, and covered to reduce evaporation; and that all pipe shall be Grade A steel, mill-tested to 1 100 lb pressure.

An area of 1 600 acres would be protected in this manner by the project, all of which is located in the "high risk zone," where Forest Service officials expect 90% of all fires to start. The total cost of materials and supplies required for the construction of this unit of the water system is $69 600. Commenting on the project publicly William V. Mendenhall, Supervisor of the Angeles National Forest, stated that with such facilities available, his men would almost guarantee to suppress any fire in the area with a loss of not to exceed 1 acre of water-shed.

In the higher mountains adjacent to this project, the danger of fire being started by human carelessness is reduced materially. If fires do occur, conditions are such that Forest Service officials believe smaller quantities of water will be required for their suppression. Consequently, fewer water lines are required in these areas of secondary risk, and the cost of the water system per square mile is reduced materially.

Before the disaster, the La Crescenta-Montrose water-shed included both "high" and "secondary" risk zones. A similar water system, designed to protect the entire burned area, could have been constructed at any time previous to the fire, at a cost for materials and supplies of $150 000. The cost of relief labor, or that of labor by the Civilian Conservation Corps, could not be considered as a true charge against the project. However, if constructed by contract, the installation of all facilities necessary for fire protection would not have cost as much as the removal of débris immediately following the disaster.

Unquestionably, sites for débris basins should be purchased and other protective measures, as outlined by Mr. Eaton, constructed for the protec-

tion of communities from débris flow in case the water-shed is denuded by fire. It would also seem logical to spend an equivalent sum of money to insure against such a catastrophe, by the installation of water systems and other facilities necessary to combat, successfully, the many forest fires that do occur. This is especially true when the chances are 100 to 1 that, sooner or later, the water-shed will be destroyed unless such facilities are provided.

R. W. DAVENPORT,[10] M. AM. SOC. C. E. (by letter).[10a]—In studying this paper, the writer has been interested in examining, rather critically, the section entitled "Flood Hydrograph Determination", and some of the incidental observations are submitted herewith.

Fig. 3 shows the run-off for the basin for the years ending September 30, 1903 to 1930, plotted against the respective indices of seasonal wetness, or the estimated mean annual precipitation over the basin expressed in percentage of the mean as determined for the period, 1872–1930. The run-off from 1916 to 1930 was estimated from the record of the Sunland gauging station of the U. S. Geological Survey evidently by application of the drainage area ratio (124.5 sq miles ÷ 106 sq miles), and, from 1903 to 1915, the run-off was deduced from the record of the neighboring San Gabriel Basin. These data give the twenty-eight points plotted in the diagram.

In undertaking to express a general relationship between precipitation and run-off in a problem of this kind, it is a commonly accepted procedure to consider run-off as a residual after the precipitation has been subjected to losses by evaporation and transpiration, and possibly other losses, usually of relatively minor importance. These losses, like the precipitation, are thus to a large extent the resultant of the operation of meteorological agencies. It is not practicable, of course, to determine the factor representing the losses by direct observation, but it may be determined indirectly by subtracting the run-off of a basin from the precipitation for periods selected so as to eliminate or to minimize differences due to changes in surface or ground-water storage. It is the writer's observation that for humid regions the factor varies in a rather definite and systematic manner, dependent upon latitude, altitude, temperature, and other regional characteristics and that from year to year it has an amplitude of variation corresponding roughly to that for precipitation. In arid regions, where the rainfall is insufficient to meet the potential demands of evaporation, transpiration, etc., the losses, of course, will tend to be smaller and more variable than in regions where the rainfall is ample.

Table 5 has been examined with regard to the indicated losses from 1903 to 1930, and the results, for groups determined by the amount of the annual precipitation, are listed in Table 14. Over the period of years considered, these data show a definite tendency for the losses in the Big Tujunga Basin to increase as the precipitation increases. For rainfalls of 30.1 in., and more, the losses varied from 22.5 to 32.7 in. and averaged 27.4 in.—apparently not much different from what would be expected in any similarly located humid region of similar temperature and similar quantities of rainfall.

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[10] Hydr. Engr., U. S. Geological Survey, Washington, D. C.

[10a] Received by the Secretary October 14, 1935.

The curve drawn through the plotted points of Fig. 3 takes an upward direction, so that for an annual precipitation of 54.6 in. (index of seasonal wetness = 200), the losses would approximate 23.4 in., whereas for the maximum year (1883–84), with a precipitation of 65.8 in., the application of the derived relationship by extrapolation, gives losses amounting to only 18.1 in. However, the facts indicated by Table 14 and the interpretation of

TABLE 14.—PRECIPITATION, RUN-OFF AND LOSSES, BIG TUJUNGA DRAINAGE BASIN

Range of precipitation, in inches	Number of years in groups	Mean precipitation, in inches	Mean losses, in inches
20 and less	7	16.9	16.2
20.1–30.0	9	24.1	21.4
30.1 and more	12	35.7	27.4

general experience elsewhere, as noted, suggest that the losses with 65.8 in. of precipitation would be materially more than 18.1 in. and might well be assumed to be 27 in., or more. This assumption would lead to a reduction of the estimated seasonal run-off for the maximum year from 316 700 acre-ft to probably not more than 250 000 acre-ft. In view of the rather large difference thus obtained, the writer would be pleased to have the observations of the author regarding it.

The writer is especially interested in the forms of hydrographs shown in Fig. 6, because they seem to depart from the forms that he would expect from his experience with comparable streams. The hydrographs seem to show that for four successive days, substantially the entire run-off from rainfall on each day cleared the basin within a 24-hr period. General experience and a superficial examination of the rainfall and run-off conditions of the Big Tujunga Basin suggest that the run-off from the rainfall on a given day will continue in substantial, but gradually decreasing, amounts for two or three days after the time of maximum flow from that rainfall. Studies of characteristics of streams in this respect have been presented by LeRoy K. Sherman, M. Am. Soc. C. E.,[11] and Merrill M. Bernard, M. Am. Soc. C. E.[12] Therefore, the writer is considerably interested to know whether there are conditions in the Big Tujunga Basin that warrant the assumption of hydrographs as shown, which are seemingly somewhat at variance with the indications of experience elsewhere.

The writer has not examined what, if any, difference these considerations might make in the conclusions with regard to flood flows. In general, a momentary peak of 388 cu ft per sec per sq mile for the Big Tujunga drainage basin as adopted seems a reasonable and appropriately conservative estimate for the peak flow of a rare flood.

The compilation of data and experience on flood and erosion control problems in this paper present a most interesting and valuable contribution.

[11] "Streamflow from Rainfall by Unit-Graph Method", *Engineering News-Record*, April 7, 1932.

[12] "An Approach to Determinate Stream Flow", *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 347.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

DISCUSSIONS

ADAPTATION OF VENTURI FLUMES TO FLOW MEASUREMENTS IN CONDUITS

Discussion

BY MESSRS. N. F. HOPKINS, AND HUNTER ROUSE

N. F. HOPKINS,[15] M. AM. SOC. C. E. (by letter)[15a].—The experiments reported by the authors are interesting as confirming the formulas. Especially interesting are the experiments showing that the velocity is uniform throughout the cross-section of the flume at such comparatively low velocities.

It would seem as if the theory could have been set forth in a more simple manner. The velocity increases with the drop in the surface from the back-water to the flume, whereas the area of the cross-section decreases with the drop. The quantity is the product of the area and the velocity. If there is no drop, there would be no velocity and quantity; and, if the drop is a maximum, there would be no area and no quantity. At some point, the product of the velocity and the area is a maximum.

For example, in addition to the notation of the paper, let p = the pitch of the sides of the flume; Z = the elevation of the back-water above the bottom of the flume plus the velocity head at back-water, minus the friction loss in entering the flume; H_v = the velocity head in the flume; d = the depth of water in the flume $= Z - H_v$ (see Fig. 11); V = the velocity in the flume $= \sqrt{2 g H_v}$; and A = the area of the water section in the flume. Furthermore, let $a = B d - A$, then;

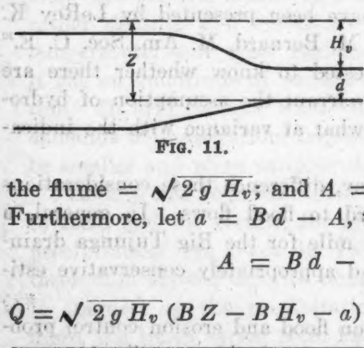

FIG. 11.

$$A = B d - a = B Z - B H_v - a \quad \text{.............} (22)$$

$$Q = \sqrt{2 g H_v}\,(B Z - B H_v - a) = \sqrt{2 g}\,(B Z H_v^{0.5} - B H_v^{1.5} - a H_v^{0.5}) \quad (23)$$

NOTE.—The paper by Harold K. Palmer, M. Am. Soc. C. E., and Fred D. Bowlus, Assoc. M. Am. Soc. C. E., was published in September, 1935, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[15] Civ. and Min. Engr. (Harrop & Hopkins), Pittsburgh, Pa.

[15a] Received by the Secretary October 11, 1935.

and for a maximum value of Q:

$$\frac{dQ}{dH_v} = \sqrt{2\,g}\left(\frac{B\,Z}{2\,H_v^{0.5}} - \frac{3\,B\,H_v^{0.5}}{2} - \frac{a}{2\,H_v^{0.5}}\right) = 0 \quad\text{........(24)}$$

In Equation (24), the quantity in parenthesis equals zero, and, consequently, $B\,Z - 3\,B\,H_v - a = 0$; that is:

$$H_v = \frac{Z}{3} - \frac{a}{3\,B} \quad\text{........................(25)}$$

For a rectangular flume, $a = 0$ and Equation (25) becomes $H_v = \frac{Z}{3}$ and Equation (23) becomes:

$$Q = 3.09\,B\,Z^{1.5} \quad\text{........................(26)}$$

For a V-shaped flume, in which $a = \frac{B(Z - H_v)}{2}$, Equation (25) is:

$$H_v = 0.2\,Z \quad\text{........................(27)}$$

and, Equation (23) is,

$$Q = 2.297\,p\,Z^{2.5} \quad\text{........................(28)}$$

For a trapezoidal flume, in which $a = (Z - H_v)^2\,p$, Equation (25) becomes:

$$H_v = \frac{0.3\,b}{p} + 0.6\,Z - \sqrt{\left(\frac{0.3\,b}{p} + 0.6\,Z\right)^2 - \frac{Z}{5}\left(\frac{b}{p} + Z\right)} \quad\text{...(29)}$$

and Equation (22) becomes:

$$A = b\,(Z - H_v) + p\,(Z - H_v)^2 \quad\text{..............(30)}$$

Substitute the value of H_v found by Equation (29) in Equation (30) to find A; and in Equation (23) to find Q. An approximate expression for the flow is:

$$Q = 3.06\,(b + 0.72\,p\,Z)\,Z^{1.5} \quad\text{..................(31)}$$

For a parabolic flume, in which $a = \frac{B}{3}\,(Z - H_v)$, Equation (25) becomes:

$$H_v = 0.25\,Z \quad\text{........................(32)}$$

Equation (22) becomes:

$$A = \frac{B\,Z}{2} \quad\text{........................(33)}$$

and Equation (23) becomes:

$$Q = 2\,B\,Z^{1.5} \quad\text{........................(34)}$$

Finally, it is necessary to find the value of B for $d = 0.75\,Z$.

For a circular flume (or, in fact, for a flume of any cross-section), it is sufficiently exact to assume that,

$$Q = 4 A \sqrt{Z} \qquad (35)$$

in which A is the area of a section of the flume, the depth of which is 0.75 Z.

HUNTER ROUSE,[16] ESQ. (by letter).[16a]—In so far as the subject-matter specified in the title of this paper is concerned, the writer must commend the authors for their able solution of a practical hydraulic problem. The fact is not to be disputed that any appropriate measuring device may be expected to yield satisfactory results once its rating curve has been determined directly or indirectly by volumetric measurement. Hence, the authors were fully justified in choosing a measuring device which eliminated every possible objectionable feature, in adapting it to the needs of the situation, and then rating it in place through comparison with a device of known characteristics. In view of the accuracy required (an allowable error of perhaps 5%, or more), it was not amiss to apply certain empirical checks upon the characteristics of flow encountered, and to express the flow equations in simplified form through several common approximations.

Approximate methods, however, are truly justifiable only if the investigator realizes the fact that the assumptions made do not fully describe the actual state of flow. When the authors claim that their paper "presents the theoretical hydraulic principles involved" in the performance of the Venturi flume, the writer believes that certain supplementary remarks have a definite place in this discussion. For although the authors state that "the formulas developed for the flume are not empirical but are based entirely on theory", the analysis of flow which they have presented is really only the first page of a long and complex story.

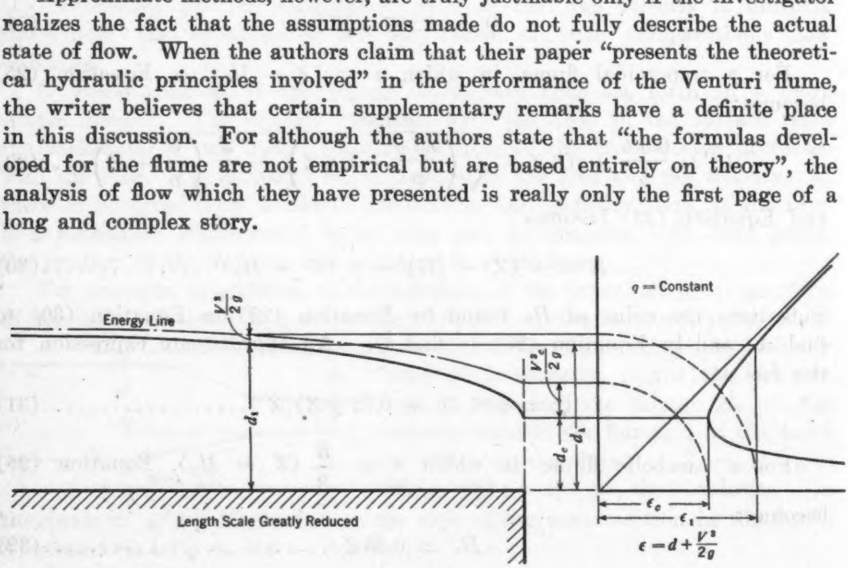

FIG. 12.

The specific energy diagram used by the authors applies correctly only to flow in long open channels in which the gradual change in surface profile is entirely the result of energy loss, and not of horizontal or vertical contraction of the flow section. Thus, in Fig. 12 is shown the surface curve in a long

[16] Instr. in Civ. Eng., Columbia Univ., New York, N. Y.

[16a] Received by the Secretary October 28, 1935.

channel of mild slope ending in an abrupt fall, the specific energy of flow, and hence, also, the depth of flow, decreasing in the direction of motion; the specific energy must reach a minimum value, for the given rate of discharge, in the vicinity of the fall, in which region the depth is equal to the critical. It must be remembered that such an energy diagram is constructed on the assumption that the flow at every section is so nearly parallel that accelerative forces can be ignored; that is, the pressure distribution is considered static throughout, so that the sum of pressure head, $\frac{p}{\gamma}$, and elevation, z, must always equal the depth of flow. The vertical distance between the energy line and the free surface must then be equal to the velocity head.

The authors' equations for flow through the Venturi flume, however, are practically identical with those used in the customary treatment of the broad-crested weir, in which the change in surface profile is entirely the result of contraction of the flow section (see Fig. 13). Not only is the flow assumed parallel throughout, but the further assumption is made that no energy loss occurs. Hence, instead of the specific energy diagram, recourse must be taken to the discharge curve, which considers the discharge per unit width as a function of the depth for a given energy line elevation. In the case of two-dimensional motion over the broad-crested weir, since $\epsilon = \frac{V^2}{2g} + d$, the unit discharge may then be expressed as:

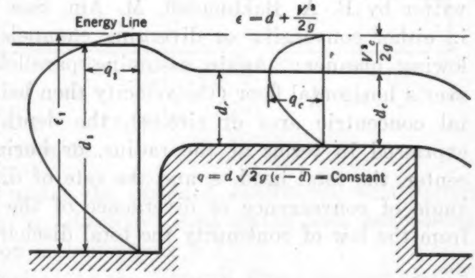

Fig. 13.

$$q = \frac{Q}{b} = V d = d \sqrt{2 g (\epsilon - d)} \quad \text{..............(36)}$$

Taking the derivative of q with respect to d,

$$\frac{\delta q}{\delta d} = \sqrt{2 g} \left(\sqrt{\epsilon - d} - \frac{d}{2 \sqrt{\epsilon - d}} \right)$$

and setting this equal to zero $\left(\frac{2 \epsilon - 3 d}{2 \sqrt{\epsilon - d}} = 0 \right)$; and:

$$d_c = \frac{2}{3} \epsilon = \sqrt[3]{\frac{q^2}{g}} \quad \text{......................(37)}$$

Obviously, the critical depth, d_c, derived by this method must be identical with that used by the authors, although the original premises are quite different.

From Equation (36) it is apparent that q must be zero when $d = 0$ and when $d = \epsilon$, and will attain its maximum value when $d = d_c$. Two such q-curves are plotted in Fig. 13, the water surface over the entire length of the weir coinciding with the point of maximum discharge on the smaller q-curve. For any other depth of flow the rate of discharge must be less than the maximum on the corresponding q-curve as shown in the region of approach.

As stated by the authors, a single measurement of depth in the throat should suffice for the determination of the discharge under these conditions, according to Equation (37). As also pointed out by the authors, a definite relationship must exist between the critical depth and the depth of approach, so that once this relationship is known a single measurement of the latter depth would also be sufficient.

A slightly different use may be made of the discharge curve in a procedure that is more directly applicable to the Venturi flume. As suggested to the writer by B. A. Bakhmeteff, M. Am. Soc. C. E., the surface curve for flow in either converging or diverging channels may be approximated in the following manner: Again assuming parallel flow at constant specific energy over a horizontal floor (the velocity then being normal at all points to horizontal concentric arcs of circles), the depth of flow at any section may be expressed in terms of the radius, or horizontal distance, r, from the fixed center, the total head, ϵ, and the rate of discharge per unit width, q. If the angle of convergence or divergence of the vertical walls is designated by θ, from the law of continuity the total discharge will be:

$$Q = \theta\, r\, V d = \theta\, r\, q = \text{constant} \qquad (38)$$

Introducing the discharge per unit width at the critical section (where q must again be a maximum):

$$\theta\, r\, q = \theta\, r_c\, q_c \text{ and } \frac{q}{q_c} = \frac{r_c}{r} \qquad (39)$$

Since,

$$\epsilon = \frac{V^2}{2\,g} + d = \frac{3}{2}\, d_c = \text{constant} \qquad (40)$$

$$q = V\, d = d\, \sqrt{2\,g\,(\epsilon - d)} \qquad (41)$$

and at the critical section,

$$q_c = \sqrt{g}\; d_c^{\frac{3}{2}} = \sqrt{g}\left(\frac{2}{3}\,\epsilon\right)^{\frac{3}{2}} \qquad (42)$$

Substituting Equations (41) and (42) in Equation (39):

$$2.6\, \frac{d}{\epsilon}\, \sqrt{1 - \frac{d}{\epsilon}} = \frac{r_c}{r} = \frac{q}{q_c} \qquad (43)$$

Equation (43) is thus a dimensionless relationship between the ratio of depth to total head and either the ratio of the critical radius to the radius at

any section or the ratio of the unit discharge at any section to the critical unit discharge. This equation will yield the q-curve and the two surface curves shown in Fig. 14. It will be noted that in the region between $r = 0$

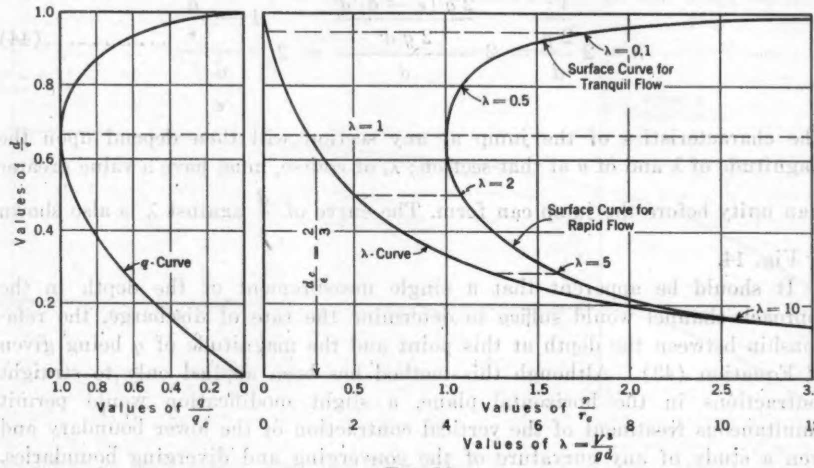

FIG. 14.

and $r = r_c$ no flow can occur under the assumed conditions, this region being known in hydrodynamics as a "singular" zone which does not belong to the region in which the function is continuous.

These relationships may be applied to the Venturi flume having vertical side contractions by assuming, as in the case of the broad-crested weir, that

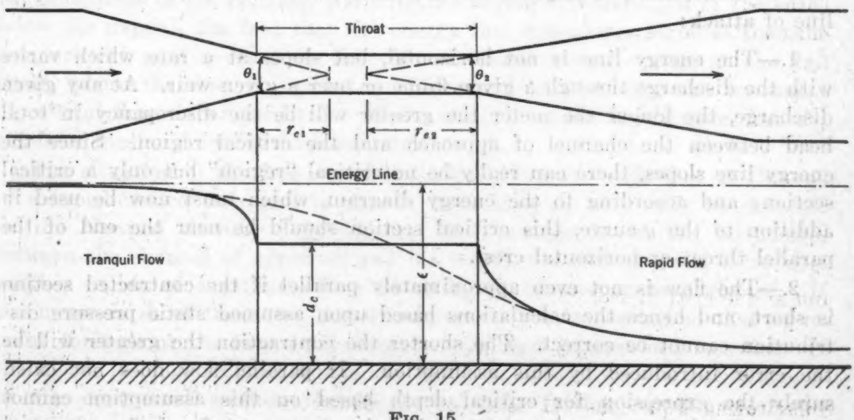

FIG. 15.

the critical depth occurs over the entire parallel section of the throat. Choosing arbitrarily the throat proportions shown in Fig. 15, this method would yield the profile given in the illustration. It is clear, as noted by the authors, that the hydraulic jump may form at any section beyond the throat, if the

tail-water level is raised, for in this region the flow is in the rapid state. Thus, the Froude number, identical with Bakhmeteff's kinetic flow factor, λ, will be:

$$\lambda = 2 \frac{\frac{V^2}{2g}}{d} = 2 \frac{\frac{2g(\epsilon - d) d^2}{2 g d^2}}{d} = 2 \frac{1 - \frac{d}{\epsilon}}{\frac{d}{\epsilon}} \qquad (44)$$

The characteristics of the jump at any section will then depend upon the magnitude of λ and of θ at that section; λ, of course, must have a value greater than unity before the jump can form. The curve of $\frac{d}{\epsilon}$ against λ is also shown in Fig. 14.

It should be apparent that a single measurement of the depth in the approach channel would suffice to determine the rate of discharge, the relationship between the depth at this point and the magnitude of q being given by Equation (43). Although this method has been applied only to straight contractions in the horizontal plane, a slight modification would permit simultaneous treatment of the vertical contraction of the lower boundary and even a study of any curvature of the converging and diverging boundaries. It is presumed, however, that the foregoing example will suffice to illustrate the point in question.

Although this procedure is fully as justifiable as the customary elementary treatment of flow over the broad-crested weir, the resulting picture of flow in the throat region is, to say the least, quite hypothetical. Actually, neither of the two basic assumptions made is completely fulfilled in the case of either weir or flume, and two major errors are thereby introduced by this elementary line of attack:

1.—The energy line is not horizontal, but slopes at a rate which varies with the discharge through a given flume or over a given weir. At any given discharge, the longer the meter the greater will be the discrepancy in total head between the channel of approach and the critical region. Since the energy line slopes, there can really be no critical "region" but only a critical section; and according to the energy diagram, which must now be used in addition to the q-curve, this critical section should be near the end of the parallel throat or horizontal crest.

2.—The flow is not even approximately parallel if the contracted section is short, and hence the calculations based upon assumed static pressure distribution cannot be correct. The shorter the contraction the greater will be the error introduced by this assumption. If parallel flow does not exist, surely the expression for critical depth based on this assumption cannot apply. Thus, the authors' statement that the sharp-crested weir is a "control section through which water flows at critical depth" is a fallacy, since the sharp-crested weir is the shortest possible contraction one can imagine and hence involves an extreme degree of departure from the basic assumption of parallel flow.

In order to clarify the error introduced by this assumption, consider the region of the fall shown in distorted scale in Fig. 12. If the length scale is given its correct proportion to the vertical scale, the profile of flow will be that shown in Fig. 16. As the flow approaches the abrupt drop, vertical

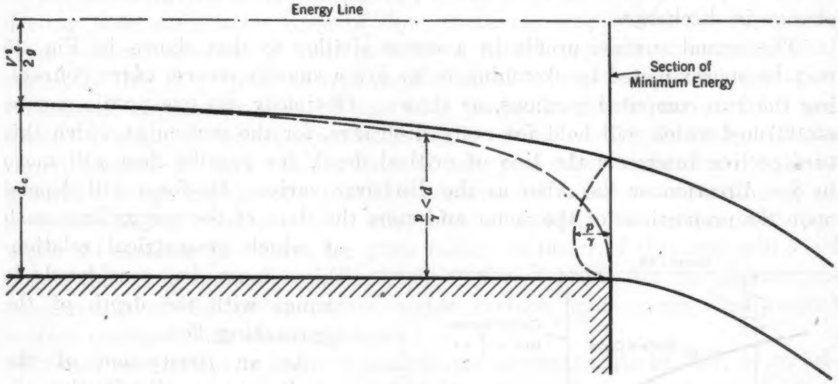

FIG. 16.

acceleration will begin, and the nappe will be deflected in the downward direction. Since curvature of the filaments requires the action of centripetal forces, the pressure will no longer be statically distributed, and the pressure head will be less than the depth below the free surface by an amount depending upon the elevation and curvature of the filaments. Although the computed critical depth for parallel flow may still be found some distance up stream from the fall (a distance varying with the unit discharge, with the slope and roughness of the channel, and with the degree of ventilation of the space below the nappe), the fact that the energy line still slopes signifies that the true critical section—that is, that of minimum energy—exists at the fall itself, where the flow is highly curvilinear. Unfortunately, the treatment of critical flow under conditions of non-static pressure distribution has not yet been mastered by the hydraulician.

The situation at the free over-fall is closely parallel with that at the end of a broad-crested weir and is also an indication of what may be expected in the Venturi flume. Furthermore, the convergence of the boundaries between the channel of approach and the contracted section itself introduces another region of curvilinear flow which by no means can be neglected. If the meter is short these two transition regions partly overlap so that at no point throughout the contraction can parallel flow be assumed to exist. If the meter is long, reaction waves are likely to occur over the entire length of the contracted section, thereby introducing further curvature of the filaments.

These two basic fallacies in the customary elementary presentation thus present a combination of difficulties which can be ignored only in an approximate analysis, for it is almost impossible to devise a meter which, for a large discharge range, will be both short enough to make the energy loss

inappreciable and long enough to insure nearly parallel flow at some definite section of the contraction. Not only will the true relationship between the actual depth of approach and the theoretical critical depth have to be determined by laboratory or field calibration, but the critical section for parallel flow—if it exists at all—will shift in position along the crest or throat with change in discharge.

The actual surface profile in a meter similar to that shown in Fig. 15 may be approximated by sketching in by eye a smooth reverse curve connecting the two computed portions, as shown. Obviously, no one profile can be established which will hold for every discharge, for the section at which this surface line intersects the line of critical depth for parallel flow will move in one direction or the other as the discharge varies. Its form will depend upon the proportions of the meter and upon the slope of the energy line, each of which geometrical relationships must be considered to change with the depth of the approaching flow.

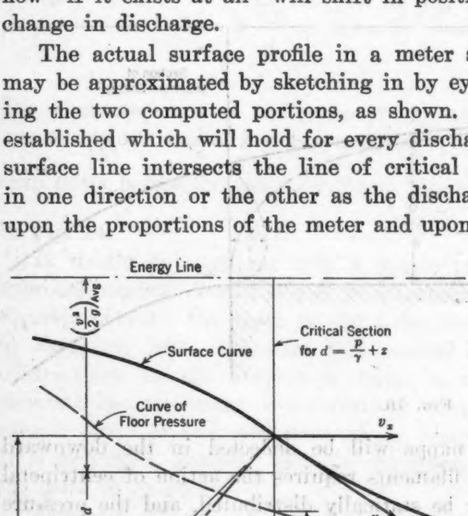

FIG. 17.

As an illustration of the actual energy distribution in the throat of the meter, reference is made to Fig. 17, in which it is assumed, for the time being, that the total head is the same at all depths of flow. Although the depth at this section happens to be exactly equal to the computed critical depth, this really has no significance whatsoever, since the pressure distribution is not static; on the contrary, the pressure head, $\left(\frac{p}{\gamma}\right)$, at any elevation, z, above the floor, is considerably less than the distance, $(d - z)$, below the free surface, as a result of the vertical acceleration of the fluid particles at this section. Although the average velocity of flow as found from the equation of continuity, $V = \frac{q}{d} = \frac{1}{d}\int^{d} v_x \, \delta d$, is commonly used in the Bernoulli theorem, the latter may be written properly only in terms of the velocity vector, v, at any point and not its component, v_x, in the direction of flow; thus the true velocity head at any elevation, for the conditions shown, is everywhere greater than the assumed mean value, with the sole exception of the point at the water surface itself. In general, it may be said that in curvilinear flow both $\frac{v^2}{2g}$ and $\frac{p}{\gamma}$ will vary with z, so that the correct

form of the Bernoulli theorem must be the following:

$$\epsilon = \frac{v^2}{2\,g} + \frac{p}{\gamma} + z \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (45)$$

To the foregoing discrepancies in the assumptions traditionally made in dealing with such basic types of flow, there must be added further errors arising from variation in total head from floor to surface; from additional curvature due to vertical as well as horizontal contraction; from separation (change of actual flow section) and eddy formation due to abrupt inlet and excessive angle of divergence; and, finally, from partial drowning of the throat because of back-water. Even if many of these difficulties could be avoided through the use of a carefully constructed flume, attempts to produce a "critical-depth" meter of this kind with a constant theoretical coefficient under all passable discharges are quite futile; no meter of this type will yield a flow that is dynamically similar under different heads, for the geometrical relationship of the meter profile, water surface, and energy line cannot remain constant with changing depth.

When an attempt is made to analyze the physical state of flow, as undertaken by the authors, it is impossible to remain content with approximate methods; furthermore, it is not at all justifiable to claim theoretical completeness when it has not been attained. If the essential accuracy of the authors' experimental work is indicated by their practice of measuring static pressure closely enough simply by turning a Pitot tube sidewise, it is obvious that further refinement is quite needless; but if hydraulicians intend to use the Venturi flume in the future for more fundamental research, it is to be hoped that they will not follow blindly such approximate and empirical paths.

One must not conclude from these pages that the simple methods of attack are without value. On the contrary, if the full significance of the elementary facts contained in this discussion were more fully realized, the Boussinesq number would not have appeared in hydraulic literature on Venturi flumes, nor would much of the search for a true critical depth meter have been undertaken.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

STABILIZING CONSTRUCTED MASONRY DAMS BY MEANS OF CEMENT INJECTIONS

Discussion

BY JAMES B. HAYS, M. AM. SOC. C. E.

JAMES B. HAYS,[9] M. AM. SOC. C. E. (by letter).[9a]—This is an unusually clear and detailed exposition of the general subject of grouting. The author has given, or outlined, a routine to follow in grouting that will apply to most conditions, particularly in foundation work. Variations in the behavior of different holes develop almost immediately since no two holes are alike. The operator or engineer must "feel" his way, depending on the pressure developed and the rate of flow of the grout which can be varied by changing the mixture.

Equipment for grouting has been developed gradually from makeshift apparatus into machinery specially suited for the work. Pumping grout has generally replaced the practice of forcing it in by compressed air. Engineers and operators who have worked with both systems prefer the continuous flow and positive drive as given by the pump, rather than the intermittent method with the possible danger of getting air into the hole. Mr. Cole has given an excellent description of the equipment, but the writer would like to know more about the pistons and cylinder liners.

Although regular cements are the most generally used, the writer, after handling approximately 40 000 bags of cement screened through a No. 200 screen, is inclined to think that it has some advantages, one of which is that the cement remains in suspension for a longer time and does not clog fittings as quickly. Certainly, small cracks and seams are more likely to be grouted with the coarse particles removed than if they are left in or, to carry it a step further, if sand is added. The writer is strongly opposed to the use of sand in grouting except in extreme cases and will admit that the leakage into the reservoirs through the up-stream faces could probably be stopped by no other method. Sand has a tendency to settle particularly when used with thin

NOTE.—The paper by D. W. Cole, M. Am. Soc. C. E., was published in February, 1935, *Proceedings*. Discussion on the paper has appeared in *Proceedings*, as follows: August, 1935, by Messrs. Oren Reed, F. F. Fergusson, and Joseph Wright; September, 1935, by Charles W. Comstock, M. Am. Soc. C. E.; and October, 1935, by V. L. Minear, Assoc. M. Am. Soc. C. E.

[9] Engr., U. S. Bureau of Reclamation, Denver, Colo.

[9a] Received by the Secretary September 16, 1935.

mixtures. Very little cement, if any, remains with it to form a tight filler or to bond the particles together. Instead, it will be found in a porous condition. Many grouting jobs have been only partly done by the inclusion of sand. When the top of a concrete-lined tunnel is filled, with sand in the grout, it will carry a load but cannot always be considered to be water-tight. The writer would suggest that before grout is used with sand, test cylinders be made using the proposed mix. Naturally, very fine sands will remain in suspension longer than coarse grades.

In the case cited by the author the leaks were low down, and the sand could be depended upon to settle to the low point. If leaks are higher, or if water is washing grout away as fast as it is pumped in, a floating substance such as sawdust often does the work of stopping the leak. Rolled oats, bran (mill feed), and other substances can be used until the leak is stopped, after which straight cement is used to the point of refusal.

When leaks are on the surface and are known in advance, the writer has found it convenient to patch a considerable part of the leak before grouting is begun, leaving small outlets at frequent intervals for the escape of water, air, and thin grout. The final sealing of the patch can then be completed in short order. Various materials are used in patching, such as wedges, oakum, strips of cloth, lead wool, and quick-setting cements. The writer has had better results with a quick-setting cement mixed so as to give a flash set, and applied to the leak in a dry form. The leak furnishes sufficient moisture to cause the mix to set. Where cracks are wide, they are generally caulked with oakum, lead wool, or other material first, after which a quick-setting cement mixture is applied as soon as the leaks appear.

It is convenient, and a widespread practice, to designate the grout mixtures by the water-cement ratio (by volume). A water meter reading directly in cubic feet is useful for this purpose. Cement is generally handled and measured in bags. Grout can be mixed, using any number of full bags of cement, and the water can be metered to any fraction of a cubic foot.

Ordinarily, the writer favors isolated grouting and would prefer to drill a hole and grout it before the other holes in the vicinity were drilled. Usually, the specifications require that, when grout leaks through adjacent open holes, they be capped and also that the capped hole be grouted as soon as possible. The writer has seen grout leak from several holes, and it was absolutely impossible to start grout into any of the leaking holes before the cement had begun to set. As a consequence, the entire area was only partly grouted. Filling a group of holes simultaneously is not satisfactory since each hole takes grout at different pressures and each may require a different mixture. However, the writer wonders what the difference is between, say, four 25-ft holes and one 100-ft hole. Each of the 25-ft holes would take grout differently and, certainly, each 25-ft section of the 100-ft hole has different characteristics. This is an argument for "stage" grouting as well as for drilling single holes and grouting them before other near-by holes are drilled. Then, again, there are exceptions to this rule.

If the nature of the rock is such that there are seams carrying a material that could be washed out and replaced with grout, it would probably be advis-

able to drill about three holes and wash between them. The efficiency of the washing is generally doubtful. The writer has spent days trying to wash clay from a seam in the foundation of the Calderwood Dam, in Tennessee. After a few hours the water would run clear. If tested again the following day the seam would be found as dirty as ever. After several days of this experience, the holes were grouted regardless. The theory was that the clay was tight and solid in the joint and would resist leakage from the reservoir. This dam was completed in 1930 and five years later there was no increase in the originally very small leakage.

The writer has grouted more than one hole at the same time where underground connections required that it be done, by feeding small batches to each hole in turn, rotating from hole to hole until all were grouted to refusal. This method lacks the advantages of continuous flow that is one of the features of pumping grout. It can be compared with the intermittent batch system that has largely been abandoned. Considerable pains must be taken to obtain the proper mixture in each hole where variations in the mix are required.

The description of the author's experience in using the "lubrication" method and the chemical grouting is appreciated. It would seem that every effort was made to get the best possible results from these methods; hence the conclusions derived have the highest value.

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DISCUSSIONS

THE HYDRAULIC JUMP IN TERMS OF DYNAMIC SIMILARITY

Discussion

BY I. M. NELIDOV, ASSOC. M. AM. SOC. C. E.

I. M. NELIDOV,[51] ASSOC. M. AM. SOC. C. E. (by letter).[51a]—A graphical interpretation of the experimental results pertaining to the determination of the length of a hydraulic jump is presented in this paper. The actual depths of flow varied from 0.032 ft to 0.253 ft and the corresponding lengths of the jump, from 1.62 ft (average of the values in Table 1) to 2.50 ft.

Two questions arose in the writer's mind relative to these experiments. The first was, whether the experiments with small depths and volumes apply to the depths measured in feet and to quantities amounting to hundredths of cubic feet per second per second. The situation with respect to weirs shows that, notwithstanding the laws of similarity, the discharges of full-sized weirs cannot be estimated with a reasonable degree of accuracy. It appears that some other factors have a preponderant influence when the scale of a phenomenon is increased. These factors may be the air entrained in water flowing with high velocity, different roughness conditions, pulsation of flow, assymmetry in conditions of approach, and, perhaps, some others.

Another question was whether it is entirely correct to designate as the length of a jump—the length of its top roller, which ordinarily is longer than the length of the transition from one stage to another. For the most part, the surface curve of this transition is concave upward and only near the conjugate depth, d_2, does it acquire a sharp curvature concaved downward.[52] This indicates that the velocities of a high order may occur farther down stream than is ordinarily anticipated.

NOTE.—The paper by Boris A. Bakhmeteff, M. Am. Soc. C. E., and Arthur E. Matzke, Jun. Am. Soc. C. E., was published in February, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1935, by Hunter Rouse, Esq.; May, 1935, by Messrs. Sherman W. Woodward, Robert E. Kennedy, L. Standish Hall, and Morrough P. O'Brien; August, 1935, by Messrs. F. V. A. E. Engels, Baldwin W. Woods, and J. C. Stevens; and September, 1935, by Messrs. Nolan Page, Andrei I. Ivanchenko, and F. T. Mavis and A. Luksch.

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[51a] Received by the Secretary October 14, 1935.

[52] "Ueber den Heutigen Stand des Wasserbaulichen Versuchswesen", von Prof. E. Meyer-Peter, *Schweizeriche Bauzeitung*, February 11, 1922, Fig. 3.

As a matter of reference the writer wishes to mention a study made by V. I. Aravin[53] based on the theorem of Bernouilli and using the computation of losses occurring in the top roller as a means for determining its length. The results were plotted on a curve for $\frac{d_2}{d_c}$ against $\frac{L}{d_c}$, L being the length of the roller, for the limits, 1.5 to 3.5 and 0.4 to 1.6, correspondingly. This curve agrees well with the experiments by Bakhmeteff, Pietrowsky, Safranez, Einwachter, and Aravin, plotted on the aforementioned curve.

[53] "The Determination of the Length of a Hydraulic Jump", by V. I. Aravin, *Transactions*, Scientific Research Inst. of Hydrotechnics, Vol. XV, Leningrad, 1935.